Math 4160 - Final Exam

Name:____

1. Prove: If the list $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is linearally dependent then one of the vectors is a linear combination of the other two vectors.

2. Suppose V and W are finite dimensional vector spaces. Assume $T \in \mathcal{L}(V, W)$. Prove if T is injective then Nullity(T) = 0.

3. Let e_1, e_2, e_3 be an orthonormal basis for an inner product space V. Prove

$$\|a_1e_1 + a_2e_2 + a_3e_3\| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

4. Let $v_1 = (1, 2, 0), v_2 = (0, 2, 0)$ and $v_3 = (2, -1, 0) \in \mathbb{R}^3$. Use Gram-Schmidt Orthoganilzation to find an orthonoral normal basis.

5. Find $U^\perp$ for $U = \{x\} \subseteq \mathcal{P}_2(\mathbb{R})$ with the inner product below

$$\langle f, g \rangle = \int_0^1 fg dx.$$

6. Let $T \in \mathcal{L}(V)$ so that

- $T^2 v = Tv$ for all $v \in V$ and
- every vector in Null(T) is orthogonal to every vector in the Range(T)
  .

Show that T is a projection on some set U.

7. Assume P_U is an orthogonal projection of U in inner product space V. Prove the $\text{Null}(P_U) = \text{Range}(I - P_U)$.

8. Define $T : \mathbb{R}^2 \to \mathbb{R}^3$ where $T(x, y) = (x, x - y, x + y)$. Compute $\text{Null}(T')$.