Math 4160 - Quiz 5

Name:____

For the following show all work clearly.

1. Find the eigenvalues and bases of eigenspaces (the eigenvectors) for

$$A = \begin{bmatrix} 4 & -5 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

2. Rewrite the following in terms of $\langle u, v\rangle$ and $\|u\|$ and $\|v\|$. Make certian to justify each step.

$$\langle 2v - 4u, v + 3u\rangle$$

3. Prove Theroem 6.1.1 parts a and c from your book.

4. Answer the questions for the standard inner product defined on $\mathcal{P}_2$.
   (a) $\|x^2\|$
   (b) $d(x^2, 1 - x - x^2)$
   (c) the angle θ between x^2 and $1 - x - x^2$.

5. Answer the questions for the inner product defined on $\mathcal{P}_2$.

$$\langle f, g\rangle = \int_{-1}^{1} f g \, dx$$

   (a) $\|x^2\|$
   (b) $d(x^2, 1 - x - x^2)$
   (c) the angle θ between x^2 and $1 - x - x^2$.

6. Answer the questions for the evaluation inner product defined on $\mathcal{P}_2$ using the points $x_0 = 1, x_2 = 2$, and $x_3 = -1$.
   (a) $\|x^2\|$
   (b) $d(x^2, 1 - x - x^2)$
   (c) the angle θ between x^2 and $1 - x - x^2$.

7. Sketch the unit circle for $\langle (x_1, y_1), (x_2, y_2)\rangle = 3x_1x_2 + 3y_1y_2$ on $\mathbb{R}^2$.

8. Use GS Orthogonaliztion to find an ON basis for the following.

(a) $\{(1,3),(-1,2)\}$ on the inner product space defined in problem 7.

(b) $\{1, 2x^2, x-1\}$ on the inner product space defined in problem 4.

(c) $\{1, 2x^2, x-1\}$ on the inner product space defined in problem 5.

(d) $\{1, 2x^2, x-1\}$ on the inner product space defined in problem 6.

9. Find $W^{\perp}$ for the following subspaces W with the defined inner products.

   (a) $W = \{(x,y,z) \in \mathbb{R}^3 | 3x + 2y = z\}$ with the Euclidean inner product (the usual dot product).

   (b) $W = \{p(x) \in \mathcal{P}_2 | p'(x) = 0\}$ with the integral inner product defined in Problem 5.