**Math 4160 - Quiz 4**

**Name:**_____

For the following show all work clearly.

1. Calculate the column spaces and the null spaces for the following linear transformations. Also identify the domain space, the codomain space and the range space.

   (a) $A = \begin{bmatrix} 2 & 0 & 2 & 6 \\ 0 & -1 & 1 & 6 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 7 \\ 0 & -1 & 2 & 3 \end{bmatrix}$

   (b) $D : \mathcal{P}_3 \to \mathcal{P}_2$ where D is the derivative (hint: write as a matrix).

   (c) $T : \mathcal{P}_3 \to \mathcal{P}_3$ given by $T(p) = (x+1)p''$. Hint: write as a matrix.

2. Make three examples of a 3x3 matrices with a Null space that is a point, a plane and a line.

3. Write the definition of a linear transformation.

4. Prove that the null space is a linear subspace.

5. Prove that the column space is a linear subspace.

6. State the Rank Plus Nullity Theorem.

7. For the following system of linear equations.

   (a) Solve using row reduction. Write down the solution set . What is the dimension of the solution set?

   (b) Write as a matrix equation, $A\mathbf{x} = \mathbf{b}$.

   (c) For the $A$ above find the column space and the null space. Verify the rank plus nullity theorem.

   (d) What do the answers in Problem 7a and Problem 7c have to do with each other?

$$\begin{cases} x_1 - 2x_2 - 2x_4 + x_5 = 4 \\ x_3 - 2x_4 + x_5 + x_6 = 1 \\ + x_5 = 0 \end{cases}$$