Math 4160 - Quiz 3

Name:_____

For the following show all work clearly.

1. For the following lists of vectors show the list is independent or show it is dependent by displaying a non-trivial linear combination of the vectors equal to zero.
   (a) $(1,0,0,3),(0,1,0,-1),(1,2,,0,1) \in \mathbb{R}^4$.
   (b) $x^2+3x, x^2+6, x-2 \in \mathcal{P}_2$.
   (c) $x^3+3, x^2-1, x^3+2x^2+1 \in \mathcal{P}_3$.
2. Define a basis for a vector space.
3. Prove $\mathbf{v}_1 = (1,2,3)$, $\mathbf{v}_2 = (4,5,6)$ and $\mathbf{v}_3 = (7,8,9)$ is not a basis for $\mathbb{R}^3$.
4. Prove $\mathbf{v}_1 = (1,2,3)$ and $\mathbf{v}_2 = (4,5,6)$ are independent. And select another vector in $\mathbb{R}^3$ call it $\mathbf{w}$ so that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{w}$ is a basis for $\mathbb{R}^3$. Show this.
5. Compute the dimension of the vector space spanned by $\mathbf{v}_1 = (1,2,3)$, $\mathbf{v}_2 = (4,5,6)$ and $\mathbf{v}_3 = (7,8,9)$.
6. Compute the dimension of the vector space $\mathbb{C}^2$ over $\mathbb{R}$. And the dimension of the vector space $\mathbb{C}^2$ over $\mathbb{C}$. Hint first find a basis for each.
7. Let $B_1 = \{(1,2),(3,4)\}$ and $B_2 = \{(1,-1),(2,0)\}$. And let B be the standard basis for $\mathbb{R}^2$.
   (a) Find the following change of basis matrices: $P_{B_1 \to B_2}$, $P_{B_2 \to B}$ and $P_{B \to B_2}$.
   (b) Let $(2,3)$ be a vector given in the standard basis. Find the coordinates in basis B_1 and the coordinates in basis B_2. Demonstrate this by showing computing linear combination of vectors of in the bases.
8. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ be a dependent list of vectors in a vector space V where
$$\mathbf{v}_1 + 3\mathbf{v}_2 - 7\mathbf{v}_3 + 0\mathbf{v}_4 = \mathbf{0}.$$
   Prove the $\text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4) = \text{Span}(\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$.