Math 4160 - Quiz 2

Name:____

For the following show all work clearly.

1. Prove Theorem 4.1.1
2. In class we claimed that $\mathbb{C}$ is a vector space over $\mathbb{R}$. Prove the following properties 2, 6 and 7.
3. In class we claimed that $\mathbb{R}$ is **not** a vector space over $\mathbb{C}$. Show this is true by showing that $\mathbb{R}$ over $\mathbb{C}$ violates some property on the list.
4. We know $\mathbb{C}$ is a vector space over $\mathbb{R}$. Show using the two step subspace test that $W = \{3a + ai | a \in \mathbb{R}\}$ is a subspace of $\mathbb{C}$.
5. We know $\mathbb{C}$ is a vector space over $\mathbb{C}$. Show using the two step subspace test that $W = \{3a + ai | a \in \mathbb{R}\}$ is **not** a subspace of $\mathbb{C}$.
6. Show that $W = \{(a, 3a, a + b) | a, b \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.
7. Show that $W = \{(a + 1, 3a, a + b) | a, b \in \mathbb{R}\}$ is **not** a subspace of $\mathbb{R}^3$.
8. Write down the definition for a list of vectors to be **independent**. And then prove that

   A list of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots, \mathbf{v}_n$ is independent if and only if the only solution to

   $$a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 + \cdots + a_n\mathbf{v}_n = 0$$

   is the trivial solution (which is $a_1 = a_2 = \cdots = a_n = 0$).
9. Show each of the following lists are independent or dependent

   (a) $(1, 2, 3)$, $(1, 0, -1)$ in $\mathbb{R}^3$

   (b) $(1, 2, 3)$, $(4, 5, 6)$, $(7, 8, 9)$ in $\mathbb{R}^3$

   (c) x, $1 + x$, $1 - x$ in $\mathcal{P}_2$