

Math 3520 - Test 2

Name: _____

Your answers must be in clear and complete proofs.

1. Prove the set of irrationals is uncountable.

2. Let $a, b, c \in \mathbb{Z}$ with $b \neq 0$. Show if $a|b$ and $b|c$ then $a|c$.

3. Let $a, b, c \in \mathbb{Z}$. If $a|bc$ and $\gcd(a, b) = 1$ then $a|c$.

4. For the following pair of numbers, find their gcd and find a linear combination of the numbers equal to their gcd.

$$a = 253 \text{ and } b = 207$$

5. Prove $3|n^3 - n$ for every integer n .

6. Do **one** of the following. We define the algebraic structure $(\mathbb{Z}, \circledast)$ where

$$a \circledast b = ab + a + b - 2.$$

- (a) Show $(\mathbb{Z}, \circledast)$ does not satisfy G1.
- (b) Show $(\mathbb{Z}, \circledast)$ does not satisfy G2.

7. For the groups $(\mathbb{Z}_4, +)$ to $(\mathbb{Z}_5^*, \cdot)$ calculate the following orders of elements.
- Find the orders of $1, 2, 3 \in \mathbb{Z}_4$ and
 - Find the orders of $1, 2, 3 \in \mathbb{Z}_5^*$.

8. Find an isomorphism from $(\mathbb{Z}_4, +)$ to $(\mathbb{Z}_5^*, \cdot)$. Verify it is an isomorphism by computing and comparing
- $f(3 + 1)$ and $f(3) \cdot f(1)$
 - $f(2 + 2)$ and $f(2) \cdot f(2)$

9. Define $T = \{3n | n \in \mathbb{Z}\}$. Clearly T is a nonempty subset of $\mathbb{Z}$. Show using the two step subgroup test that $(T, +)$ is a subgroup of $(\mathbb{Z}, +)$.