

Math 3520 - Test 1

Name: _____

1. Prove for all $n \in \mathbb{N}$ that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}.$$

2. Prove If $n \in \mathbb{Z}$ and n is odd then $4|n^2 - 1$.

3. Prove: If A is well ordered and $B \subseteq A$ then B is well ordered.

4. We define the given relation on $A = \{1, 2, 3, 4\}$ by

$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 4), (4, 1), (3, 4), (4, 3), (a, b)(c, d)\}$$

- (a) Assume R is an equivalence relation. Find (a, b) and (c, d) .
- (b) What are the equivalence classes for R .

5. Prove the relation defined below on $\mathbb{Z}$ is an symmetric and transitive.

$$a\mathcal{R}b \iff 4|a + 3b$$

6. Let $f : A \rightarrow B$. Prove if f and g are injective then $g \circ f$ is injective.

7. For the following permutaions:

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$

compute:

(a) $\sigma_1 \circ \sigma_2$.

(b) σ_3^3 .

(c) σ_4^{-1} .

(d) How many elements in S_4 ?

8. Prove $(1, 3) \sim (-4, 7)$.