Math 3520 - Final Exam Review

Prepare

- Test 1
- Test 2
- Test 2 Review
- and this Review

1 New Stuff

1. Define $\lim_{x \to c} f(x) = L$
2. Compute and prove the following limits
   (a) $\lim_{x \to 2} 3x + 2$
   (b) $\lim_{x \to -3} x^2 + 1$
   (c) $\lim_{x \to -1} 2x^2$
   (d) $\lim_{x \to 2} x^3$
   (e) $\lim_{x \to -2} x^3 + 1$

2 Group Theory

3. Definition of a **subgroup** and of an **isomorphism**.
4. For the group $(\mathbb{Z}, +)$ prove, using the 2-step subspace test, that $H = \{3n : n \in \mathbb{Z}\}$ is a subgroup.
5. For the group $(\mathbb{Z}, +)$ Show $H = \{2n + 1 : n \in \mathbb{Z}\}$ is not a subgroup.
6. For the group $(S_3, \circ)$ Show

$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \right\}$$

   is not a subgroup of S_3.

7. Let G be any group and let g be a fixed element of G then prove, using the 2-step subspace test, that $H = \{gag^{-1} | a \in G\}$ is a subgroup.

8. Let G be any group then prove, using the 2-step subspace test, that $H = \{a \in G | ag = ga \, \forall g \in G\}$ is a subgroup.

9. For the groups $G_1 = (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, +)$ and $G_2 = (\mathbb{Z}_8, +)$

   (a) Find the orders of the elements $(1, 1, 1)$ and $(1, 0, 1)$ in G_1 and the orders of the elements 1, 2, 3 in G_2?

   (b) Are any of the above elements generators for their respective groups?

   (c) Why aren't the groups isomorphic?

10. For the groups $G_1 = (\mathbb{Z}_9^*, \cdot)$ and $G_2 = (Z_6, +)$

    (a) Find the orders of three different non identity elements in G_1 and the orders of the elements 1, 2, 3 in G_2?

    (b) Are any of the above elements generators for their respective groups?

    (c) The two groups are isomorphic. Find the isomorphism $f : G_1 \to G_2$.

11. Note that $(G, \cdot)$ is a group where $G = \{2^n : n \in \mathbb{Z}\}$ and $\cdot$ is regular multiplication. Prove Axioms G_1 and G_3 for $(G, \cdot)$.

12. Note that $(G, \cdot)$ is a group where $G = \{2^n : n \in \mathbb{Z}\}$ and $\cdot$ is regular multiplication. Show $G \cong \mathbb{Z}$ where $\mathbb{Z}$ is a group over addition. I used the isomorphism $f : \mathbb{Z} \to G$. We need to prove f preserves the operation and that f is a bijection.

3 Functions

13. Define $f : \{1, 2, 3\} \to \{4, 7, 9\}$ by $f(1) = 4$, $f(2) = 4$ and $f(3) = 9$.

    (a) Is f injective, surjective or bijective? Compute

    (b) Compute $f(\{1, 2\})$, $f^{-1}(\{1, 4\})$ and $f \circ f^{-1}(\{4, 9\})$.

14. Define $f : (-\infty, 0) \to [0, \infty)$ by $f(x) = x^2$.

    (a) Is f injective, surjective or bijective? Prove or disprove.

(b) Compute $f((-2,2))$, $f^{-1}((-2,2))$, $f^{-1}\circ f(\{4,9\})$ and $f\circ f^{-1}(\{4,9\})$.

15. Define $f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \setminus \{1\}$ by $f(x) = \frac{x}{x-2}$. Is f injective, surjective or bijective? Prove or disprove.

16. Let $f : A \to B$ and $g : B \to C$. Prove the following.

    (a) If f and g are injective then $g \circ f$ is injective.

    (b) If f and g are surjective then $g \circ f$ is surjective.

4 Cardinality

17. Define cardinality. That is $A \sim B$ if and only if ...

18. Know $\sim$ is an equivalence relation and what that means.

19. Show $\mathbb{N} \sim 2\mathbb{N}$

20. Show $\mathbb{N} \sim \mathbb{Z}$

21. Let $A = [0,2]$ and $B = [-1,6]$. Show $f : A \to B$ given by $f(x) = \frac{7}{2}x - 1$ is a bijection. What does this tell uas about the sets A and B?