Math 3520 - Quiz 5

Name:____

Type proof in complete and proper English.

1. State and prove the cancellation law.
2. Are the following algebraic structures groups? If it is a group prove it. If not prove it is not a group.
   (a) $(\mathbb{Q}^*, \cdot)$ where $\cdot$ is regular multiplication.
   (b) $(\mathbb{Z}, \odot)$ where $a \odot b = a + b - 2$.
   (c) $(\mathbb{Z}, \circledast)$ where $a \circledast b = ab + a + b$.
   (d) $(\mathbb{Q} \setminus \{-1\}, \circledast)$ where $a \circledast b = ab + a + b$.
3. For the following algebraic structures write the operation tables. State if the structure is a group or not. If not state which property fails and how. Identify the identity in each case.
   (a) $(\mathbb{Z}_5, +)$
   (b) $(\mathbb{Z}_5, \cdot)$
   (c) $(\mathbb{Z}_6^*, \cdot)$
   (d) $(D, \circ)$ where

$$D = \{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\}$$

4. Prove the identity of a group is unique.
5. Prove. Let G be a group. If $a \in G$ then a^{-1} is unique.
6. If $a, b \in G$ then $(ab)^{-1} = b^{-1}a^{-1}$.