Math 3520 - Quiz 4

Name:____

Type proof in complete and proper English.

1. Define $f : \mathbb{Z}_6 \to \mathbb{Z}_3$ by $f(n) = n \mod 3$ Is f injective, surjective or bijective? Prove or disprove.

2. Define $f : (0, 4) \to (1, 13)$ by $f(x) = 3x + 1$. Prove f is bijective.

3. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2 + 1$. Show f is not injective and show f is not surjective.

4. Let $f : A \to B$ and $g : B \to C$. Prove the following.
   (a) If f and g are injective then $g \circ f$ is injective.
   (b) If f and g are surjective then $g \circ f$ is surjective.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - 1$. Compute
   - $f((-3, 2))$
   - $f^{-1}((-3, 2))$
   - $f \circ f^{-1}((-3, 2])$
   - $f^{-1} \circ f((-3, 2))$

6. Let $f : A \to B$. Prove the following.
   (a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$ for any sets $A_1, A_2 \subseteq A$.
   (b) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$ for any sets $A_1, A_2 \subseteq A$.
   (c) Demonstrate that the $\subseteq$ in Problem 6b cannot be equality by defining a function $f : A \to B$ and two sets $A_1, A_2 \subseteq A$ so that $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$.

7. List all elements of the set $\mathcal{S}_3$. How many elements are in the set $\mathcal{S}_6$?

8. For the following permutaions:
   $\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$
   compute:
   (a) $\sigma_1 \circ \sigma_1$
   (b) $\sigma_1 \circ \sigma_2 \circ \sigma_2$
   (c) σ_1^3
   (d) σ_1^{-1}