

Math 3520 - Quiz 3

Name: _____

Type proof in complete and proper English.

1. If A has the Well Ordered Property and B is a nonempty subset of A then B has the well ordered property.
2. Show $7|4^{3n} - 1$ for all $n \in \mathbb{N}$. Use induction.
3. Let $A = \{1, 2, 3\}$ And let

$$R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 3)\}$$

be a relation on A . What is the domain of R What is the range of R ?
Find R^{-1} .

4. Let $A = \{1, 2, 3\}$ And let

$$R = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 3)\}$$

be a relation on A .

- (a) Is R reflexive? Prove or disprove.
 - (b) Is R symmetric? Prove or disprove.
 - (c) Is R transitive? Prove or disprove.
5. Let $a, b \in \mathbb{Z}$ and let

$$aRb \Leftrightarrow a - b \leq 2$$

be a relation on $\mathbb{Z}$.

- (a) Is R reflexive? Prove or disprove.
- (b) Is R symmetric? Prove or disprove.
- (c) Is R transitive? Prove or disprove.

6. Let $a, b \in \mathbb{Z}$ and let

$$aRb \Leftrightarrow 3|a - b$$

be a relation on $\mathbb{Z}$. Prove R is an equivalence relation on $\mathbb{Z}$. And determine the distinct equivalence classes.

7. ~~Write down the definition of a partition.~~

8. Prove the following theorem:
Theorem : Let A a set and let R be an equivalence relation on A .
 Prove that the equivalence classes of R forms a partition of A . Hint
 you may use the lemma from class.
9. Let R_1 and R_2 be equivalence relations on the set A . Prove that $R = R_1 \cap R_2$ is also an equivalence relations on the set A .
10. Construct the addition and multiplication tables for $\mathbb{Z}_4$ and $\mathbb{Z}_5$.
11. Three questions nearly identical.
 - (a) Let $[a], [b] \in \mathbb{Z}_8$. If $[a][b] = [0]$ then does it follow that either $[a] = 0$ or $[b] = [0]$.
 - (b) Let $[a], [b] \in \mathbb{Z}_9$. If $[a][b] = [0]$ then does it follow that either $[a] = 0$ or $[b] = [0]$.
 - (c) Let $[a], [b] \in \mathbb{Z}_7$. If $[a][b] = [0]$ then does it follow that either $[a] = 0$ or $[b] = [0]$.