1 From earlier

1. Let $\mathbf{r}(t) \to \mathbb{R} \to \mathbb{R}^3$ be differentiable so that the $\mathbf{r}$ has a constant norm.

   (a) Prove that the path $\mathbf{r}$ and its derivative are perpandicular.

   (b) Come up with a nonzero example of $\mathbf{r}(t)$.

2. Let $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$.

   (a) Prove the parallelogram rule

   $$\|\mathbf{v}+\mathbf{w}\|^2 + \|\mathbf{v}-\mathbf{w}\|^2 = 2\|\mathbf{v}\|^2 + 2\|\mathbf{w}\|^2$$

   (b) graph some vectors $\mathbf{v}$, $\mathbf{w}$, $\mathbf{v}+\mathbf{w}$ and $\mathbf{v}-\mathbf{w}$ in $\mathbb{R}^2$ to explain the parallelogram rule.

2 Paths

3. Let $\mathbf{r}(t) = \langle t^2 - t, t^3 - 3t^2 + 3\rangle$

   (a) Find the position, velocity and acceleration of the particle at time $t = 2$.

   (b) Graph the position, velocity and acceleration appropriately.

4. Let $\mathbf{r}(t) = \langle \sin(e^{-t}), \cos(e^{-t})\rangle$

   (a) Find the speed function
   $$\frac{ds}{dt} = \|\mathbf{r}'(t)\|$$

   (b) Compute the arc length from $t = 0$ to $t = 1$.

   (c) Compute the arc length from $t = 0$ to $t = \infty$.

   (d) What is the graph of $\mathbf{r}(t)$?

5. Let $\mathbf{r}(t) = \langle e^{-t}\sin(t), e^{-t}\cos(t)\rangle$

   (a) Find the speed function
   $$\frac{ds}{dt} = \|\mathbf{r}'(t)\|$$

   (b) Compute the arc length from $t = 0$ to $t = 1$.

   (c) Compute the arc length from $t = 0$ to $t = \infty$.

   (d) What is the graph of $\mathbf{r}(t)$?

3 Functions of Several Variables

6. Compute the following limits if they exist. If not show why.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2+1}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2}$

   (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{x^2+y^2}$

7. Let $f(x,y) = x^2+y^2$. Consider the points $P(0,2)$ and $Q(1,2)$.

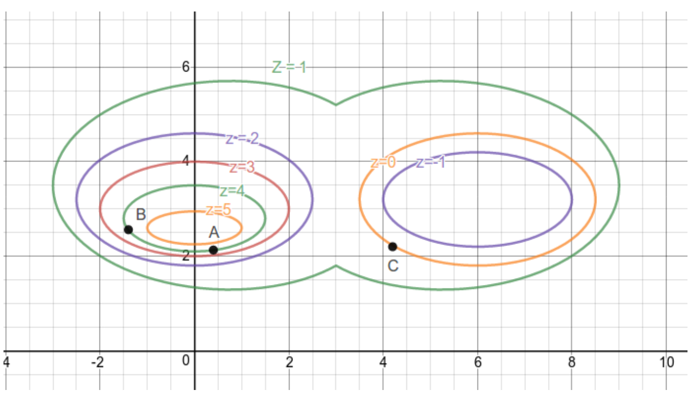
   (a) Graph the contour plot. Include $z = -1, 0, 1, 2, 3, 4$.

   (b) Compute the $\nabla f(x,y)$

   (c) Compute the $\nabla f(P)$ and $\nabla f(Q)$. Also compute their norms.

   (d) Graph $\nabla f(P)$ and $\nabla f(Q)$ with initial points P and Q respectively.

8. Draw the gradient at each point A, B and C.



9. Fill in the blanks.

   (a) The gradient is the direction of __________ increase.

   (b) The gradient is __________ to the contour lines.

   (c) The norm of the gradient is __________.

   (d) A larger norm of one gradient is __________ graphically.

10. Let $f(x,y) = e^{xy^2+2} - xy^3 + 2$. Find the tangent plane to $f(x,y,z)$ at the point $(-2,1)$. Use that plane to estimate $f(-2.1, 0.8)$. Compare to the real value of $f(-2.1, 0.8)$.

11. Let $f(x,y,z) = x^2y - xy^2 + z^3$. Find the tangent plane (actually a hyperplane) to $f(x,y,z)$ at the point $(1,2,3)$

12. Let $A(1,2,3)$ and $B(1,1,2)$ be points in $\mathbb{R}^3$. Let $\mathbf{v} = \langle 1,2,3\rangle$ and $\mathbf{w} = \langle 2,2,4\rangle$ and $f(x,y,z) = x^2 + y^2 + z^2$.
    (a) Compute $\nabla f(A)$ and $\|\nabla f(A)\|$.
    (b) Compute $D_{\mathbf{v}} f(B)$ and $D_{\mathbf{w}} f(B)$. One is bigger than the other. Interpret.
    (c) Compute $D_{\mathbf{v}} f(A)$. Compare to $\|\nabla f(A)\|$.
13. Let $f(x,y) = x^3 + x^2y^2$. Let $x(t) = t^2 + 1$ and $y(t) = 2t - 1$.
    (a) Use the chain rule to compute $\frac{df}{dt}$.
    (b) Compute $\frac{df}{dt}$ at $t = 1$.
    (c) Compute the point given in the path at $t = 1$.
    (d) Let $\mathbf{v} = \langle 2,2\rangle$. Compute $D_{\mathbf{v}} f(2,1)$. Is this question familiar to you?
14. Find and classify extremma.
    (a) $f(x,y) = x^2 - xy + y^3$.
    (b) $f(x,y) = x^2 + 2xy - y^4$.
    (c) $f(x,y,z) = x + 3y + z$ subject to $x^2 + y^2 + z^2 = 1$.
    (d) $f(x,y,z,w) = x^2 + y^2 + z^2 + w^2$ subject to $x + y + z - 3w = 4$.
    (e) $f(x,y,z,w) = x\ln(x) + y\ln(y) + z\ln(z)$ subject to $x + y + z = 1$. In this problem f is called information entropy.

4 Integrals

15. $\displaystyle\iint_R x + y\,dA$ over the region defined by $x + y = 2$ and the coordinate axes.
16. $\displaystyle\iint_R xy\,dA$ over the region defined by $y = x^2$ and the line $y = x + 1$.
17. $\displaystyle\iint_R e^{x^2}\,dA$ over the region defined by $y = -x$, $y = 2x$ and the vertical line $x = 4$.
18. $\displaystyle\iint_R e^{x^2+y^2}\,dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ in the third quadrant.
19. $\displaystyle\iint_R \sqrt{\frac{\tan^{-1}(y/x)}{x^2 + y^2}}\,dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ above the lines $y = -x$ and $y = x$.
20. Find the volume below the paraboloid $z = 12 - x^2 - y^2$ and above the xy-plane.

21. $\iint_R \sin(x-y)\cos(x+y)\,dA$ over the region defined the lines $y = x+2$, $y = x+4$, $y = -x$ and $y = -x+3$. Hint the change of variables is $u = x-y$ and $v = x+y$.

22. $\iint_R \frac{x-y}{2x+y}\,dA$ over the region defined the lines $y = x+2$, $y = x$, $y = -2x+2$ and $y = -2x+3$.

23. $\iint_R xy\,dA$ over the region defined the graphs of $xy = 1$, $xy = 3$ and the lines $y = x$ and $y = 3x$. Hint $x = u/v$ and $y = v$.

24. $\iint_R (x-y)e^{x^2-y^2}\,dA$ over the region defined the lines $y = x+2$, $y = x$, $y = -x$ and $y = -x+3$.

25. $\iint_R e^{x^2+4y^2}\,dA$ over the region defined by the portion of the ellipse $\frac{x^2}{4} + y^2 = 1$ in the third quadrant. Hint use the change of variables $x = 2v\cos(u)$ and $x = v\sin(u)$. And note I had $\pi \leq u \leq \frac{3\pi}{2}$

5 Line Integrals

26. $\int_C x\,dx$. Let C be line segment from $(0,1)$ to $(3,2)$.

27. $\int_C xy\,ds$. Let C be line segment from $(0,1)$ to $(3,2)$.

28. $\int_C \langle -x, y\rangle \cdot d\mathbf{r}$. Let C be line segment from $(0,1)$ to $(3,2)$.

29. $\int_C x\,dy$. Let C be line segment from $(0,1)$ to $(3,2)$.

30. $\oint_C xy\,dx$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

31. $\oint_C \langle -x, y\rangle \cdot d\mathbf{r}$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

32. $\oint_C \langle 1, xy\rangle \cdot d\mathbf{r}$. Let C be the circle $x^2 + y^2 = 4$ traced counter-clockwise.

33. $\oint_C -x + yds$. Let C be the circle $x^2 + y^2 = 4$ traced counter-clockwise.

6 Green's Theorem

34. $\oint_C \langle x, -y \rangle \cdot d\mathbf{r}$. Let C be outside of the square traced from $(0,0)$ to $(0,2)$ to $(1,2)$ to $(1,0)$ and then back to $(0,0)$.

35. $\oint_C \langle e^{x^3} - xy, e^{y^3} - y \rangle \cdot d\mathbf{r}$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

36. $\oint_C \langle \cos(x^2)+y, \cos(y^2)+xy \rangle \cdot d\mathbf{r}$. Let C be the circle $x^2+y^2=4$ traced counter-clockwise.

7 Div/Grad/Curl for Winter Break

37. Define
$$f(x,y,z) = x^3 - yz^2 \text{ and } \mathbf{F}(x,y,z) = \langle x^3, yz^2, xy \rangle.$$
Compute the following, if possible, and if not possible state why.

(a) $\operatorname{div}(f(x,y,z))$
(b) $\operatorname{grad}(f(x,y,z))$
(c) $\operatorname{curl}(f(x,y,z))$
(d) $\operatorname{div}(\mathbf{F}(x,y,z))$
(e) $\operatorname{grad}(\mathbf{F}(x,y,z))$
(f) $\operatorname{curl}(\mathbf{F}(x,y,z))$
(g) $\nabla \cdot \mathbf{F}(x,y,z)$
(h) $\nabla \times (\nabla \cdot \mathbf{F}(x,y,z))$
(i) $\nabla \times (\nabla f(x,y,z))$