(12.)

(a) $\int x e^{2x}\,dx$

$u = x \qquad dv = e^{2x}\,dx \quad \leftarrow$ choose

$du = dx \qquad v = \frac{1}{2}e^{2x} \quad \leftarrow$ compute

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$$\int dv = \int e^{2x}\,dx$$

$$v = \int e^{u} \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int e^{u}\,du$$

$$= \frac{1}{2}e^{2x} + C$$

$u \quad v \quad - \int v\,du$

$$= x\left(\tfrac{1}{2}e^{2x}\right) - \int \tfrac{1}{2}e^{2x}\,dx$$

$$= x\left(\tfrac{1}{2}e^{2x}\right) - \tfrac{1}{2}\left(\tfrac{1}{2}e^{2x}\right) + C$$

(c) $\int x^3 e^{2x^2}\,dx$

$u = x^2 \qquad dv = x\,e^{2x^2}\,dx$

$du = 2x\,dx \qquad v = \frac{1}{4}e^{2x^2}$

$u = 2x^2$

$$= x^2\left(\tfrac{1}{4}e^{2x^2}\right) - \int\left(\tfrac{1}{4}e^{2x^2}\right)2x\,dx$$

$$= x^2\left(\tfrac{1}{4}e^{2x^2}\right) - \tfrac{1}{2}\int x e^{2x^2}\,dx$$

$$= x^2\left(\tfrac{1}{4}e^{2x^2}\right) - \tfrac{1}{2}\left[\tfrac{1}{4}e^{2x^2}\right] + C$$

(e) $\int x^2 \sin(2x)\,dx$

$u = x^2 \qquad dv = \sin(2x)\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{2}\cos(2x)$

$$= x^2\left(-\tfrac{1}{2}\cos(2x)\right) - \int\left(-\tfrac{1}{2}\cos(2x)\right)2x\,dx$$

$u = x \qquad dv = \cos(2x)\,dx$

$du = dx \qquad v = \frac{1}{2}\sin(2x)$

$$= -\tfrac{1}{2}x^2\cos(2x) + \int x\cos(2x)\,dx$$

$$= -\tfrac{1}{2}x^2\cos(2x) + x\left(\tfrac{1}{2}\sin(2x)\right) - \int \tfrac{1}{2}\sin(2x)\,dx = -\tfrac{1}{2}x^2\cos(2x) + \tfrac{x}{2}\sin(2x) + \tfrac{1}{4}\cos(2x) + C$$

12 (h) hint $u = \ln(x)$ $dv = x^3 dx$

(i) hint $u = \arctan(2x)$ $dv = dx$

13.

(a) $\int \sin^{1/2}(2x)\ \underline{\cos(2x)\,dx}$

$u = \sin(2x)$

$du = 2\cos(2x)\,dx$

$\frac{1}{2}du = \underline{\cos(2x)\,dx}$

$= \int u^{1/2}\left(\frac{1}{2}du\right)$

$= \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\ \frac{u^{1/2+1}}{1/2+1} + C = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \boxed{\frac{1}{3}\left(\sin(2x)\right)^{3/2} + C}$

(b) $\int \sin^{1/2}(2x)\ \cos^3(2x)\,dx$

$= \int \sin^{1/2}(2x)\cdot\cos^2(2x)\cdot\cos(2x)\,dx$

$= \int \sin^{1/2}(2x)\left(1 - \sin^2(2x)\right)\ \underline{\cos(2x)\,dx}$

$u = \sin(2x)$

$du = 2\cos(2x)\,dx$

$\frac{1}{2}du = \cos(2x)\,dx$

$= \int u^{1/2}(1-u^2)\cdot\frac{1}{2}du$

$= \frac{1}{2}\int u^{1/2} - u^{5/2}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2} + - \frac{u^{7/2}}{7/2}\right] + C$

$= \frac{1}{2}\left[\frac{(\sin(2x))^{3/2}}{3/2} - \frac{(\sin(2x)\quad)}{7/2}\right] + C$

13. (e) $\int \sin^2(7x)\,dx \leftarrow$ even powers use $\sin^2(\theta) = \frac{1}{2}\left[1 - \cos(2\theta)\right]$

$= \int \frac{1}{2}\left[1 - \cos(14x)\right] dx$

$= \frac{1}{2}\left[x - \frac{1}{14}\sin(14x)\right] + C$

(j) $\int \sin^{-1}(x) \cos(x)\,dx$

$u = \sin(x)$

$du = \cos(x)\,dx$

$= \int u^{-1}\,du$

$= \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sin(x)| + C.$

(k) $\int \sin^{-2}(x)\,dx = \int \csc^2(x)\,dx = -\cot(x) + C$

(14.) (c) $\int \tan(x)dx = \ln|\sec(x)| + C$ ← Now this is a formula (# 13)

(e) $\int \sec(x)\, dx = \ln|\sec(x) + \tan(x)| + C$ ← THIS TOO (# 14)

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15. (a) $\int x^3 (9 - x^2)^{1/2} dx$

$a^2 - u^2$

$a = 3 \quad u = x$

use $u = a \sin\theta$

$x = 3 \sin\theta$

$dx = 3 \cos\theta\, d\theta$

$x = 3\sin\theta$ $\quad dx$

$= \int (3 \sin\theta)^3 (9 - (3\sin\theta)^2)^{1/2}\, 3\cos\theta\, d\theta$

$= \int 27 \sin^3\theta (9 - 9\sin^2\theta)^{1/2} \cdot 3\cos\theta\, d\theta$

$= 81 \int \sin^3\theta (9(1 - \sin^2\theta))^{1/2} \cos\theta\, d\theta$

$= 81 \int \sin^3(\theta) (9 \cdot \cos^2\theta)^{1/2} \cos\theta\, d\theta$

$= 81 \int \sin^3\theta \; 3 \cdot \cos\theta \; \cos\theta\, d\theta$

$= 243 \int \sin^3\theta \cos^2\theta\, d\theta$

$= 243 \int \sin^2\theta \cos^2\theta \; \sin\theta\, d\theta$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$= 243 \int (1 - \cos^2\theta) \cos^2\theta \; \sin\theta\, d\theta$

$= -243 \int (1 - u^2)\, u^2\, du = -243 \int u^2 - u^4\, du$

15(a) CONTINUED

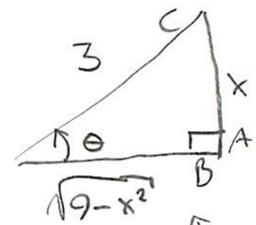
$$= -243\left[\frac{u^3}{3} + -\frac{u^5}{5}\right] + C$$

$$= -243\left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right] + C$$

$$= -243\left[\frac{\left(\frac{\sqrt{9-x^2}}{3}\right)^3}{3} - \frac{\left(\frac{\sqrt{9-x^2}}{3}\right)^5}{5}\right] + C$$

ORIGINAL SUB $x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$ $\quad \frac{opp}{hyp}$



FROM P.T.

$A^2 + B^2 = C^2$

$x^2 + B^2 = 3^2$

$B^2 = 9 - x^2$

$B = \sqrt{9-x^2}$

FROM TRIANGLE $\cos\theta = \frac{\sqrt{9-x^2}}{3}$

15. (c) $\int \frac{1}{x^2(4+x^2)^{1/2}}\,dx$

$a^2 + u^2$

$a = 2 \quad u = x$

$u = a\tan\theta$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{1}{(2\tan\theta)^2\,(4 + (2\tan\theta)^2)^{1/2}} \cdot 2\sec^2\theta\, d\theta$$

$$= 2\int \frac{1}{4\tan^2\theta\,(4 + 4\tan^2\theta)^{1/2}}\sec^2\theta\, d\theta$$

$$= \frac{2}{4}\int \frac{\sec^2\theta}{\tan^2\theta\,(4\cdot(1+\tan^2\theta))^{1/2}}\, d\theta$$

$$= \frac{1}{2}\int \frac{\sec^2\theta}{\tan^2\theta\,(4\sec^2\theta)^{1/2}}\, d\theta = \frac{1}{2}\int \frac{\sec^2\theta}{\tan^2\theta\; 2\sec\theta}\, d\theta$$

15(c)

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$= \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{1}{4}\int \frac{du}{u^2} = \int \frac{1}{4}u^{-2}\,du$$

$$= \frac{1}{4}\,\frac{u^{-1}}{-1} + C = -\frac{1}{4}\,\frac{1}{\sin\theta} + C = -\frac{1}{4}\csc\theta + C$$

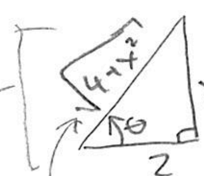
$$= -\frac{1}{4}\cdot\frac{\sqrt{4+x^2}}{x} + C$$

ORIGINAL SUB

$x = 2\tan\theta$

$\frac{x}{2} = \tan\theta$

$\csc\theta = \frac{\sqrt{4+x^2}}{x}$ $\frac{\text{HYP}}{\text{OPP}}$



$A^2 + B^2 = C^2$

$2^2 + x^2 = C^2$

$\sqrt{4+x^2} = C$