MATH 2320 Test 1

Name:_____

1. Using the **definition** of the integral compute

$$\int_0^2 2x + 3\,dx.$$

2. Let acceleration be given by $a(t) = 6t - 12e^{2t}$. And let $v_0 = 4$ and $s_0 = -3$. Find v and s. What is the velocity when $t = 3$ sec.

3. Find the area between the functions $x = y^2$, $y = x - 2$.

4. Find the volume of the solid formed when rotating the region bounded by $y = 2x$ and $y = 6 - x$ and the y-axis around the x-axis.

5. Two easy integrals:

   (a) $\int \cos(4x)\,dx$

   (b) $\int e^{-3x}\,dx$

6. $\int x^2 \ln(x)\, dx$

7. $\int x^3 \cos(x^4 + 1)\, dx$

8. $\int \sin^2(3x)\,dx$

9. $\int \sin(3x)\cos^{1/2}(3x)\,dx$

10. $\int \frac{1}{\sqrt{x^2+4}}\,dx$

take home: Zeno's Paradox
We will attempt to compute another infinite summation.

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \tag{1}$$

1. Take a guess at what the sum is in Equation 1.
2. Compute the limit $\lim_{n\to\infty} \left(\frac{1}{2}\right)^n$
3. Remember from Algebra class that

$$1 + r + r^2 + r^3 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

So for example

$$1 + 3 + 3^2 + 3^3 + 3^4 = \frac{1 - 3^{4+1}}{1 - 3} = \frac{1 - 243}{-2} = \frac{-242}{-2} = 121$$

Compute

(a) $1 + 2 + 2^2 + 2^3 + 2^4$
(b) $1 + 2 + 2^2 + 2^3 + 2^4 + \cdots + 2^{100}$ (no neeed to simplify this one)
(c) $1 + \frac{1}{2} + (\frac{1}{2})^2 + (\frac{1}{2})^3 + (\frac{1}{2})^4 + \cdots + (\frac{1}{2})^{10}$

4. What is $\lim_{n\to\infty} 1 + \frac{1}{2} + (\frac{1}{2})^2 + \cdots + (\frac{1}{2})^n$? Hint:

$$\begin{aligned}
\lim_{n\to\infty} 1 + \frac{1}{2} + (\frac{1}{2})^2 + \cdots + (\frac{1}{2})^n &= \lim_{n\to\infty} \frac{1 - (\frac{1}{2})^{n+1}}{1 - \frac{1}{2}} = \lim_{n\to\infty} \frac{1}{1 - \frac{1}{2}} - \lim_{n\to\infty} \frac{(\frac{1}{2})^{n+1}}{1 - \frac{1}{2}} \\
&= \frac{1}{1 - \frac{1}{2}} - \lim_{n\to\infty} \frac{}{1 - \frac{1}{2}} (\frac{1}{2})^{n+1} \\
&= \frac{1}{1 - \frac{1}{2}} - \frac{1}{1 - \frac{1}{2}} \lim_{n\to\infty} (\frac{1}{2})^{n+1} \\
&= \cdots \text{ you finish!}
\end{aligned}$$

5. Now you know the answer to Equation 1 (compare to your guess). Repeat your work in Question 4. Compute the following:

(a) $1 + \frac{1}{3} + (\frac{1}{3})^2 + (\frac{1}{3})^3 + (\frac{1}{3})^4 + \cdots +$
(b) $1 + \frac{-1}{3} + (\frac{-1}{3})^2 + (\frac{-1}{3})^3 + (\frac{-1}{3})^4 + \cdots +$
(c) $1 + 2 + (2)^2 + (2)^3 + (2)^4 + \cdots +$