1 Techniques of integration

1. Compute the following integrals using by parts.
   (a) $\int xe^{2x}\,dx$
   (b) $\int x^2e^{3x}\,dx$
2. Compute the following trig integrals
   (a) $\int \sin^5(3x)\,dx$
   (b) $\int \sin^2(2x)\cos^2(2x)\,dx$
3. Compute the following using trig substitutions.
   (a) $\int \frac{1}{x\sqrt{9-x^2}}\,dx$
   (b) $\int \frac{1}{(4+x^2)^{3/2}}\,dx$
4. Compute the following using partial fraction decomposition.
   (a) $\int \frac{1}{4-x^2}\,dx$
   (b) $\int \frac{2x^2-3}{4x-x^3}\,dx$
   (c) $\int \frac{3x^2-2}{4x+x^3}\,dx$
5. Compute the following improper integrals
   (a) $\int_0^1 \frac{1}{x^2}\,dx$
   (b) $\int_1^\infty \frac{1}{x^2}\,dx$
   (c) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$
   (d) $\int_0^\infty \frac{1}{\sqrt{x}}\,dx$
   (e) $\int_0^\infty \sin(x)\,dx$
   (f) $\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx$
   (g) $\int_0^\infty xe^{-x^2}\,dx$
   (h) $\int_0^\infty xe^{-x^2}\,dx$

2 Series

6. Telescoping
   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2}$

(b) $\sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$

(c) $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

7. Geometric Series

(a) $\sum_{k=1}^{\infty} 3^{-k}$

(b) $\sum_{k=1}^{\infty} \frac{1}{3^{-k}}$

(c) $\sum_{k=17}^{\infty} 11 \cdot 3^{-k}$

(d) $\sum_{k=1}^{\infty} \frac{2^k + 3^k}{4^k}$

(e) $\sum_{k=1}^{\infty} \frac{4^k}{2^k + 3^k}$

(f) $9 + -3 + 1 + -\frac{1}{3} + \frac{1}{9} + \ldots$

8. Integral Test

(a) $\sum_{k=1}^{\infty} e^{-k}$

(b) $\sum_{k=1}^{\infty} \frac{1}{1+k^2}$

(c) $\sum_{k=17}^{\infty} \frac{1}{k \ln(k)}$

(d) $\sum_{k=17}^{\infty} \frac{1}{k \ln^2(k)}$

9. P-Series Test

(a) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(b) $\sum_{k=1}^{\infty} \frac{1}{k^2}$

(c) $\sum_{k=1}^{\infty} k^3$

(d) $\sum_{k=1}^{\infty} k^{-3}$

(e) $\sum_{k=1}^{\infty} \frac{k+k^2}{k^3}$

(f) $\sum_{k=1}^{\infty} \frac{k+k^2}{k^4}$

10. Limit Comparison Test

(a) $\sum_{k=1}^{\infty} \frac{k}{k+1}$

(b) $\sum_{k=1}^{\infty} \frac{1}{3k^2+4k^3+5}$

(c) $\sum_{k=1}^{\infty} \frac{k}{3k^2+4k^3+5}$

(d) $\sum_{k=1}^{\infty} \frac{k^2}{3k^2+4k^3+5}$

(e) $\sum_{k=1}^{\infty} \frac{\sqrt{4k^3-1}}{2k+3}$

(f) $\sum_{k=1}^{\infty} \sqrt{3k+2}\frac{4k^2+2k+1}{(2k^{11}+3)^{1/3}}$

11. Ratio Test

(a) $\sum_{k=1}^{\infty} \frac{k}{2^k}$

(b) $\sum_{k=1}^{\infty} \frac{1}{k!}$

(c) $\sum_{k=1}^{\infty} \frac{2^k}{k!}$

(d) $\sum_{k=1}^{\infty} \frac{k!}{k^k}$

(e) $\sum_{k=1}^{\infty} \frac{(k!)^2}{k^k}$

(f) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

(g) $\displaystyle\sum_{k=1}^{\infty} \frac{(2k)!}{k^k}$

12. Root Test

(a) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k^2+1}{2k^2-1}\right)^k$

(b) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{3k^2+1}{2k^2-1}\right)^k$

(c) $\displaystyle\sum_{k=1}^{\infty} \left(1-\frac{1}{k}\right)^{k^2}$

(d) $\displaystyle\sum_{k=1}^{\infty} \left(1+\frac{2}{k}\right)^{k^2}$

13. Alternating Series Test

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}(-1)^k$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{k+1}(-1)^k$

(c) $\displaystyle\sum_{k=1}^{\infty} \left(1-\frac{1}{k}\right)^k (-1)^k$

3 Approximating Functions with Polynomials

14. **Approximating Functions with Polynomials**

(a) Let $f(x) = \ln(2x-3) + 2x$ find a polynomial of degree 3 that approximates $f(x)$ near the point $a = 2$. Use the polynomial to approximate $f(3)$.

(b) Let $f(x) = x^5$ find a polynomial of degree 3 that approximates $f(x)$ near the point $a = 2$. Use the polynomial to approximate $f(3)$.

4 Power Series

15. **Power Series** Find the radius of convergence and interval of convergence for the following power series.

(a) $\sum_{n=1}^{\infty} \frac{1}{n} x^n$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} x^n$

(c) $\sum_{n=1}^{\infty} \frac{n^2+1}{n+1} x^n$

(d) $\sum_{n=1}^{\infty} \frac{1}{2^n} x^n$

(e) $\sum_{n=1}^{\infty} \frac{1}{n!} x^n$

(f) $\sum_{n=1}^{\infty} \frac{3^n}{4^n+1} x^n$

(g) $\sum_{n=1}^{\infty} \frac{1}{2n} x^{2n}$

16. **Taylor Series** Find the Taylor series for the following functions centered at a

(a) $f(x) = e^{2x}$ where $a = 0$

(b) $f(x) = e^{2x}$ where $a = 1$

(c) $f(x) = e^x$ where $a = 0$

(d) $f(x) = \sin(x)$ where $a = 0$

(e) $f(x) = \cos(x)$ where $a = 0$

(f) $f(x) = \frac{1}{1-x}$ where $a = 0$

17. **Taylor Series** Find the Taylor series using known series.

(a) $f(x) = e^{2x}$ where $a = 0$

(b) $f(x) = x^2 e^x$ where $a = 0$

(c) $f(x) = \frac{e^x - 1}{x}$ where $a = 0$

(d) $f(x) = \frac{1}{1+x}$ where $a = 0$

(e) $f(x) = \frac{1}{1+x^2}$ where $a = 0$

(f) $f(x) = \arctan(x)$ where $a = 0$

18. **Conic Sections** Graph the following

(a) $x^2 + y^2 = 1$

(b) $x^2 - y^2 = 1$

(c) $-x^2 + y^2 = 1$

(d) $-x^2 - y^2 = 1$

(e) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

(f) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(g) $-x^2 + y = 1$

(h) $-x^2 - y = 1$

(i) $x - y^2 = 1$

(j) $y = x^2$