MATH 2320 Practice Test 1

1 Anti Derivative and u sub

1. Compute the following Integrals

   (a) $\displaystyle\int \sqrt{3x} + 4\sec(x)\tan(x) - \frac{2}{\sqrt{1-x^2}}\,dx$

   (b) $\displaystyle\int \frac{3x^2 - 4\sqrt{x} + 3}{x}\,dx$

   (c) $\displaystyle\int x^2\sqrt{x^3+1}\,dx$

   (d) $\displaystyle\int x^2 \sin x^3 + 1\,dx$

   (e) $\displaystyle\int x^2 \sec^2 x^3 + 1\,dx$

   (f) $\displaystyle\int \csc(x)[\cot(x) + \sin(x)]\,dx$

   (g) $\displaystyle\int \frac{x}{1+x^2}\,dx$

   (h) $\displaystyle\int \frac{x}{1+x^4}\,dx$

   (i) $\displaystyle\int \frac{e^x}{1+e^x}\,dx$

   (j) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$ Hint $\frac{e^x}{1+e^{2x}} = \frac{e^x}{1+(e^x)^2}$ and I used $u = e^x$.

2 Definition of the integral

2. Using the **definition** of the integral compute

$$\int_1^4 3x - 2\,dx.$$

3. Using the **FTC II** compute

$$\int_1^4 3x - 2\,dx.$$

4. Using the **FTC I** compute the following. Be certain to demonstrate each step.

   (a) $\dfrac{d}{dx}\left[\displaystyle\int_4^{x^2} f(t)\,dt\right]$

   (b) $\dfrac{d}{dx}\left[\displaystyle\int_4^{3x+2} e^{t^3}\,dt\right]$

   (c) $\dfrac{d}{dx}\left[\displaystyle\int_x^{2x} f(t)\,dt\right]$

3 Application of the integral

5. Velocity, Acceleration and Position

   (a) Let $a(t) = -5\sin(t)$, $v(0) = 5$ and $s(0) = -7$.

      i. Find $v(t)$ and $s(t)$.

      ii. When does the object stop?

   (b) Let $a(t) = -12t$, $v(0) = 6$ and $s(0) = 0$.

      i. Find $v(t)$ and $s(t)$.

      ii. When does the object stop?

      iii. What is the position of the object when it stops?

6. Find the area between the functions

   (a) $y = x^2$ and $y = 4$.

   (b) $y = x^2$ and $y = x + 1$.

   (c) $y = e^{3x}$, $y = 4$ and the y-axis.

   (d) $x = y^2$ and $x = 4$.

   (e) $x = y^2$ and $y = x - 1$.

   (f) $y = \ln(x)$, $x = 1$ and $y = 4$.

   (g) $y = \ln(x)$, $x = 3$ and the x-axis.

7. Find the volume of the solid formed when rotating the region bounded by $y = x^2$ and $y = 4$ around the x-axis using discs.

8. Find the volume of the solid formed when rotating the region bounded by $y = x^2$ and $y = 4$ in the first quadrant around the y-axis using discs.

9. Find the volume of the solid formed when rotating the region bounded by $y = e^{3x}$, $y = 4$ and the y-axis around the x-axis using discs.

10. Rotate the region bounded by $y = 3x$, $y = 4$ and the y-axis around the y-axis using discs.

11. Rotate the region bounded by the ellipse $x^2 + \frac{y^2}{4} = 1$, $y = 2x - 2$ around the x-axis using discs.

    (a) Set up the integral with discs.

    (b) Compute the integral.

12. Compute the following integrals using by parts.

    (a) $\int xe^{2x}\,dx$

    (b) $\int x^2e^{3x}\,dx$

    (c) $\int x^3e^{3x^2}\,dx$ Hint $u = x^2$ so what is dv?

    (d) $\int x\sin(2x)\,dx$

    (e) $\int x^2\sin(2x)\,dx$

    (f) $\int \ln(x)\,dx$

    (g) $\int x\ln(x)\,dx$

    (h) $\int x^3\ln(x)\,dx$

    (i) $\int \arctan(2x)\,dx$

13. Compute the trigonometric integrals.

    (a) $\int \sin^{1/2}(2x)\cos(2x)\,dx$

    (b) $\int \sin^{1/2}(2x)\cos^3(2x)\,dx$

    (c) $\int \sin^3(3x)\,dx$

    (d) $\int \sin^3(5x)\cos^3(5x)\,dx$

    (e) $\int \sin^2(7x)\,dx$

    (f) $\int \sin^4(7x)\,dx$

    (g) $\int \sin^2(2x)\cos^2(2x)\,dx$

    (h) $\int \sin^2(2x)\cos(2x)\,dx$

    (i) $\int \sin(x)\cos^{-1}(x)\,dx$

    (j) $\int \sin^{-1}(x)\cos(x)\,dx$

    (k) $\int \sin^{-2}(x)\,dx$

(l) $\int \cos^{-2}(x)\,dx$

(m) $\int \sin(x)\cos^{-2}(x)\,dx$

(n) $\int \sin^{-2}(x)\cos(x)\,dx$

14. There are six trigonometric functions yet we only know the integrals for a few. Complete the list.

(a) $\int \sin(x)\,dx$

(b) $\int \cos(x)\,dx$

(c) $\int \tan(x)\,dx$ Hint use Problem 13i

(d) $\int \cot(x)\,dx$ Hint use Problem 13j

(e) $\int \sec(x)\,dx$ Hint use

$$\sec(x) = \sec(x)\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)} = \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}$$

(f) $\int \csc(x)\,dx$ Imitate the work from Problem 14e but multiply by

$$\frac{\csc(x)+\cot(x)}{\csc(x)+\cot(x)}$$

(g) $\int \sec^2(3x)\,dx$

(h) $\int x\csc^2(5x^2+1)\,dx$

(i) $\int \sec(x)\tan(x)\,dx$

(j) $\int \csc(x)\cot(x)\,dx$

15. Compute the following using trig substitutions.

(a) $\int x^3\sqrt{9-x^2}\,dx$

(b) $\int \frac{x^3}{\sqrt{4-x^2}}\,dx$

(c) $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

(d) $\int \frac{1}{\sqrt{x^2-4}}\,dx$

(e) $\int \frac{x^2}{\sqrt{x^2-4}}\,dx$

(f) $\int \frac{1}{(4-x^2)^{3/2}}\,dx$

16. ~~Compute the following using partial fraction decomposition.~~

(a) ~~$\int \frac{1}{4-x^2}\,dx$~~

(b) ~~$\int \frac{2x^2-3}{4x-x^3}\,dx$~~

(c) ~~$\int \frac{3x^2-2}{4x+x^3}\,dx$~~

(d) ~~$\int \frac{3x^3-2}{4x+x^3}\,dx$~~

17. Compute the following integrals using u-sub.

(a) $\int xe^{x^2}\,dx$

(b) $\int x\sec^2(x^2)\,dx$

(c) $\int (x^2+2)e^{x^3+6x}\,dx$

(d) $\int x\sqrt{x-1}\,dx$

(e) $\int x\sqrt{x^2-1}\,dx$

(f) $\int \frac{3x}{1+x^2}\,dx$

(g) $\int \frac{4}{1+x^2}\,dx$

(h) $\int \sec^2(x)\sin(\tan(x))\,dx$

(i) $\int e^{2x}\sec(e^{2x})\tan(e^{2x})\,dx$

(j) $\int \tan(x)\,dx$. Hint use $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and u-sub.

(k) $\int \cot(x)\,dx$

(l) $\int \frac{e^x}{1+e^{2x}}\,dx$

(m) $\int \frac{e^{2x}}{1+e^{2x}}\,dx$