1 Related Rates

1. The radius of a spherical balloon is increasing at a rate of 3 inches per second. When the radius is 7 inches, how fast is the volume of the balloon increasing?

2. The radius of a spherical balloon is increasing at a rate of 3 inches per second. When the radius is 6 inches, how fast is the surface area of the balloon increasing?

3. The surface area of a spherical balloon is increasing at a rate of 10 square inches per second. When the radius is 4 inches, how fast is the volume of the balloon increasing?

4. The volume of a spherical balloon is increasing at a rate of 10 cubic inches per second. When the radius is 9 inches, how fast is the surface area of the balloon increasing?

5. See section 3.11 problems 20, 23 and 24

2 Graphing

6. For the following functions find the first derivative number line, the seconderivative number line, the extremma and the coordinates of the extremma, and inflection points (and intercepts if simple). Use information to graph the function.

   (a) $f(x) = x^3 + 9x^2 + 24x$
   (b) $f(x) = x^4 - 9x^2$
   (c) $f(x) = x^4 - 12x^3 + 9x^2$
   (d)

3 First Derivative Test

7. For the following functions find the first derivative number line, and classify the extremma. Also find the coordinates of the extremma.

   (a) $f(x) = x^3 - 12x$
   (b) $f(x) = (x^2 - 1)^4$
   (c) $f(x) = (x - 3)^3(x + 5)^4$
   (d) $f(x) = \frac{x}{1+x^2}$.
   (e) $f(x) = e^{x^2}$
   (f) $f(x) = e^{x^3-x}$
   (g) $f(x) = \sin(x)\cos(x)$ for $x \in [0, 2\pi]$

4 Second Derivative Test

8. For the following functions find all critical points and and classify the extremma using the second derivative. Also find the coordinates of the extremma.

   (a) $f(x) = xe^x$

   (b) $f(x) = \arctan(x^2)$

   (c) $f(x) = x^4 - 12x^3 + 9x^2$

   (d) $f(x) = e^{x^2}$

5 Optimization

9. Try section 4.4 problems 14, 15, 16, 17, 18

6 L'Hopital's Rule

10. Find the indicated limit.

   (a) $\lim_{x\to 3} \frac{x^2-2x+9}{x^2-9}$

   (b) $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$

   (c) $\lim_{x\to \pi} \frac{\sin(x)}{x-\pi}$

   (d) $\lim_{x\to 0} \frac{4\sin(5x)}{3x}$

   (e) $\lim_{x\to 0} \frac{\sin(x)-x+\frac{1}{3}x^3}{x^5}$

   (f) $\lim_{x\to \infty} \frac{x^3+1}{e^x}$

   (g) $\lim_{x\to \infty} \frac{3x+\sqrt{x}+1}{x}$

   (h) $\lim_{x\to 0} \frac{\tan(x)}{x}$

   (i) $\lim_{x\to 0} x^x$

   (j) $\lim_{x\to 0}(1+3x)^{\frac{1}{x}}$

   (k) $\lim_{x\to 0}(1-3x)^{\frac{1}{x}}$

   (l) $\lim_{x\to 0}(1-3x^3)^{\frac{1}{x^2}}$

   (m) $\lim_{x\to 0}(1+\sin(x))^{\frac{1}{x}}$

7 Antiderivatives

11. Compute the Antideivatives

    (a) $\int 3\cos(x) - 2\csc^2(x) + e^x - 2x\,dx$
    (b) $\int 2\sec(x)\tan(x) + \frac{3x^2-1}{x}\,dx$
    (c) $\int \frac{1}{\sqrt{1-x^2}}\,dx$
    (d) $\int 3e^x - 2x^2 + \cos(x) + \frac{1}{\sqrt{1-x^2}}\,dx$
    (e) $\int \frac{6}{x\sqrt{x^2-1}} + \frac{8}{1+x^2}\,dx$
    (f) $\int x^\pi + e^\pi\,dx$

12. Compute the Antiderivatives with substitution

    (a) $\int \sin(3x)\,dx$
    (b) $\int e^{5x+1}\,dx$
    (c) $\int \sec^2(4x)\,dx$
    (d) $\int \frac{1}{3x+1}\,dx$
    (e) $\int \tan(x)\,dx$. Hint $\tan(x) = \frac{\sin(x)}{\cos(x)}$
    (f) $\int x\sin(3x^2+1)\,dx$
    (g) $\int x\sqrt{x^2+14}\,dx$
    (h) $\int (2x1) + \sqrt{x^2+x+3}\,dx$
    (i) $\int x^2\sqrt{1-4x^3}\,dx$
    (j) $\int \sin(4x)\cos(4x)\,dx$
    (k) $\int \sin(4x)\cos^2(4x)\,dx$
    (l) $\int \sec(4x)\tan(4x)\,dx$
    (m) $\int \tan(4x)\,dx$
    (n) $\int \frac{e^x}{1+e^x}\,dx$
    (o) $\int \frac{e^x}{1+e^{2x}}\,dx$

8 Always Be Computing Derivatives (ABCD)

13. Compute the following derivatives

    (a) $f(x) = 3\sec(x) - 5\ln(x) + 1$
    (b) $f(x) = \ln(x)$

(c) $f(x) = 4\sin^2(x)\cos^2(x) + \cos^2(2x)$

(d) $f(x) = x^2(\tan(x) + 1)^3$

(e) $f(x) = \arctan(x) + e^{x^3}$

(f) $f(x) = \arctan(x^3 + 1)$

(g) $f(x) = \sin(x)\cos(x)$

(h) $f(x) = x^x$

(i) $f(x) = x^{x^x}$