

MATH 2310 Practice Test 1

1. Compute the limits

(a) $\lim_{x \rightarrow 2} x^3 + 1$

(b) $\lim_{x \rightarrow 2} \frac{x^2 - 2}{x^2 + 2}$

(c) $\lim_{x \rightarrow 2} \frac{x^2 - 2x + 1}{x^2 - 2}$

(d) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{3x}$

(e) $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}$

(f) $\lim_{x \rightarrow 0} \frac{\sin(x)}{3x}$

(g) $\lim_{n \rightarrow \infty} \frac{4n^2 + 5}{3n^2 + 6n - 5}$

(h) $\lim_{n \rightarrow \infty} \frac{n^2}{n^3 + 1}$

(i) $\lim_{n \rightarrow \infty} \frac{n^3}{n^2 + 1}$

(j) $\lim_{n \rightarrow -\infty} \frac{n^5}{n^3 + 1}$

(k) $\lim_{n \rightarrow -\infty} \frac{n^3}{n^2 + 1}$

(l) $\lim_{n \rightarrow \infty} \sqrt{2n + 1} \frac{3n + 2}{\sqrt{4n^3 + n + 11}}$

(m) $\lim_{x \rightarrow 0^+} \frac{1}{3x}$

(n) $\lim_{x \rightarrow 0^-} \frac{1}{3x}$

(o) $\lim_{x \rightarrow 0} \frac{1}{3x}$

(p) $\lim_{x \rightarrow 0^+} \frac{1}{x^2}$

(q) $\lim_{x \rightarrow 0^-} \frac{1}{x^2}$

- (r) $\lim_{x \rightarrow 0} \frac{1}{x^2}$
 (s) $\lim_{x \rightarrow 1} \frac{x+1}{x^2-1}$

2. Compute the derivative using the **definition** of the derivative

- (a) $f(x) = 3x + 5$ at $x = -1$
 (b) $f(x) = x^2$ at $x = 2$
 (c) $f(x) = x^2$
 (d) $f(x) = 5x + 1$
 (e) $f(x) = \sqrt{x}$
 (f) $f(x) = \frac{1}{x}$

3. Compute the derivative from the formulae.

- (a) $f(x) = 3x + 5\sqrt{x} - \sin(x)$
 (b) $f(x) = 5 \cos(x) - 4 \ln(x) + \sec(x)$
 (c) $f(x) = \frac{1}{x} - \sqrt{x}$
 (d) $f(x) = \frac{4-2x+2x^2}{\sqrt{x}}$
 (e) $f(x) = \frac{4\sqrt{x}-2+x^{3/2}}{\sqrt{x}}$
 (f) $f(x) = \frac{4\sqrt{x}-2}{x^{3/2}}$
 (g) $f(x) = \csc(x) - 2e^x + \cot(x)$

4. Compute the derivative from the formulae and use product rule, quotient rule and chain rule.

- (a) $f(x) = 5\sqrt{x} \sin(x)$
 (b) $f(x) = 5 \cos(x) \sec(x)$
 (c) $f(x) = 4 \ln(x) \sec(x)$
 (d) $f(x) = \ln(x^2)$
 (e) $f(x) = \ln(x^2 + 1)$
 (f) $f(x) = (x - 1)^4(3x + 2)^3$
 (g) $f(x) = \frac{3x}{\sqrt{x+4}}$
 (h) $f(x) = \frac{x}{\sqrt{x+4}}$
 (i) $f(x) = \frac{\sin(x)}{\cos(x)}$

- (j) $f(x) = \frac{e^x}{1+e^x}$
- (k) $f(x) = \frac{e^x}{1+e^{2x}}$
- (l) $f(x) = e^{e^{x^2}+1}$

5. Compute the derivative using implicit differentiation.

- (a) $x^3 + y^3 = 2x + 5$
- (b) $x^3y^3 = 2x + 5$
- (c) $\sin(x^3) + \sin(y^3) = 2x + 5$
- (d) $\sin(x^3y^3) = 2x + 5$
- (e) $\sin(xy^2) = 2x + 5y - 7$
- (f) $y = 2^x$ use logarithmic differentiation here.
- (g) $y = x^x$ use logarithmic differentiation here.
- (h) $y = x^{x^2}$ use logarithmic differentiation here.
- (i) $y = x^{2x}$ use logarithmic differentiation here.
- (j) $y = \sin(x)^x$ use logarithmic differentiation here.
- (k) $y = \sin(x)^{e^x}$ use logarithmic differentiation here.

6. Inverse Trigonometric derivatives.

- (a) $f(x) = \tan^{-1}(x)$
- (b) $f(x) = \tan^{-1}(3x^2)$
- (c) $f(x) = \sin^{-1}(e^x + 1)$
- (d) $f(x) = \sec^{-1}(\cos(x^2))$

7. Find the equation of the line tangent to $f(x) = \tan^{-1}(2e^x - 1) + x^2$ when $x = 1$.

8. Compute the first three derivatives for $f(x) = x\varepsilon^{x^2}$. I would simplify between each step. Why?

9. Let $f(x) = x^3 - 12x + 1$. Find the equation of the tangent line at $x = 1$

10. Let $f(x) = x^3 - 12x + 1$. Find the points where $f(x)$ has a zero derivative.

11. Let $f(x) = (x^2 - 1)^{3/5}$. Find the points where $f(x)$ has a zero derivative.

12. Let $f(x) = (x^2 - 1)^{3/5}$. Find the points where $f(x)$ has an undefined derivative.

13. 1 Displacement, Velocity and Acceleration

14. Let $s(t) = -4.9t^2 + 3t + 1$ represent the height of a ball we throw up in the air at time $t = 0$.

- (a) What is the height of the ball at time $t = 0$?
- (b) What is the speed of the ball at time $t = 0$?
- (c) When does the ball have a velocity of zero?
- (d) When does the ball hit the ground?
- (e) What is the velocity of the ball when it hits the ground?
- (f) Write out the velocity and acceleration equations (maybe you should do this question first).

15. Let $s(t) = 4\sin(t)$ represent the height of a ball connected to a moving spring.

- (a) What is the height of the ball at time $t = 0$?
- (b) What is the speed of the ball at time $t = 0$?
- (c) What is the height of the ball at time $t = \pi/2$?
- (d) What is the speed of the ball at time $t = \pi/2$?
- (e) When does the ball have a velocity of zero?
- (f) Write out the velocity and acceleration equations (maybe you should do this question first).

16. A rocket ship flies away from some point in space with its displacement from that point given by the equation

$$s(t) = t^3 - 3t + 7.$$

- (a) Write out the velocity and acceleration equations.
- (b) When is the ship stationary (that is, has velocity zero)?
- (c) What is the displacement at the two times from Problem 16b?

17. Related Rates. 3.11: 5, 11, 16, 17, 23, 27

18. Chapter 4

4.1: 15, 23, 27, 33, 47, 51, 76

4.2: 27, 29, 39, 41, 57, 71, 73

4.3: 9, 13, 15, 17, 50

4.4: 14, 21, 27, 28