Math 3160 - Test 2

Name:____

1. Find equation for the plane (in $\mathbb{R}^3$) so that

   (a) the plane contains the point $P(2, 2, -1)$, $Q(1, 0, 3)$ and $R(0, -1, 0)$.

   (b) the plane contains the the point $P(2, 2, -1)$ and is perpendicular to the vector $(1, -2, 0)$.

2. Let $W = \{(x, y, z) \in \mathbb{R}^3 : \text{ where } x - 3y - z = 0\}$. Use the two step subspace test to show $(W, +, \cdot)$ is a subspace.

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3. Let $S = \{(1,2,1), (0,1,2), (0,-1,0)\}$.

   (a) Is S linearlly independent? (There is an easy test for this problem).

   (b) Is $(2,2,2) \in \text{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(2,2,2)$?

   (c) Does S span $\mathbb{R}^3$?

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4. Let $B = \{(1,0),(0,1)\}$, $B_1 = \{(-1,1),(2,3)\}$ and $B_2 = \{(1,-1),(1,1)\}$.

   (a) Find the change of basis matrices for $P_{B\to B_1}$ and $P_{B_1\to B_2}$.

   (b) Find the coordinates of the point $(1,3)$ (given in the standard basis) relative to the bases B_1 and B_2.

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5. Write the matrix for the following transformations described below.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 45° counter-clockwise.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis.

   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the x-axis is contracted by half and the y-axis is dilated by 2.

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6. For the following matrices find the charactersitic equation, the eigenvalues and their cooresponding eigen vectors.

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$$
