Math 3160 - Test 1

Name:_____

No calculators and show all work.

1. Solve the following systems of linear equations using row reduction.

$$\begin{cases} x_1 - 3x_2 & & & -6x_5 = 3 \\ & +2x_3 & -4x_4 & = 6 \\ & & x_4 & x_5 = 0 \\ 2x_2 & -2x_3 & & +6x_5 = 10 \end{cases}$$

2. Solve the following systems of linear equations using row reduction.

$$\begin{cases} 2x_1 & -x_2 & +3x_3 & = 2 \\ 4x_1 & -x_2 & +x_3 & = 2 \\ x_1 & & -x_3 & = 4 \end{cases}$$

3. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

$$\begin{cases} x_1 & +2x_2 & +4x_3 & = 0 \\ x_1 & & -x_3 & = 2 \\ x_1 & +x_2 & +x_3 & = -3 \end{cases}$$

4. Solve the following using Cramer's rule.

$$\begin{cases} 3x_1 & & +4x_3 & = 0 \\ & -x_2 & +3x_3 & = 0 \\ & -3x_2 & & = 2 \end{cases}$$

5. Let T be the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ where

$$T(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} x - 2y \\ 3z + x \\ y \\ 4z \end{bmatrix}.$$

(a) Write the matrix, A, for the transformation T.

(b) Compute $T(\begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix})$.

(c) Compute $T(\begin{bmatrix} a \\ b \\ c \end{bmatrix}) + T(\begin{bmatrix} d \\ e \\ f \end{bmatrix})$

(d) Compute $T(\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix})$

6. Define the points $P(0,1,2)$, $Q(1,1,-1)$ and $R(3,0,2)$

   (a) Find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (b) Find the parametric equation of the line through points P and Q.

   (c) Find the equation of the plane through points P, Q and R. Hint the vector $(-3,-9,-1)$ is perpandicular to your plane.

7. Compute the indicated operation

(a) Compute $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}^{-2}$

(b) Compute $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}^{4}$

8. Define the planes P_1 and P_2 as follows:

$$P_1 : 3x - y + z = 12$$

$$P_2 : x - z = 4$$

(a) What are the two normal vectors for the above planes.

(b) Find the angle between the two above planes.

(c) Find set of all points that lay in both planes.