Math 4160 - Test 1

Name:____

1. Prove the following is a subspace of $V = \mathbb{R}^3$.

$$\{(x, y, z) | x + y = 0\}$$

2. Prove **one** of the following:

   (a) Let U and W be suspaces of V. Prove $U + W$ is a subspace of V.

   (b) Let U and W be suspaces of V. Prove $U \cap W$ is a subspace of V.

3. Is $(1, 0, 2, -4) \in \text{Span}((0, 0, 1, 0), (1, 1, -2, 1), (0, 1, 1, 1))$? Answer yes or no and justify your answer.

4. Prove: If the list $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is linearally dependent then one of the vectors is a linear combination of the other two vectors.

5. Suppose V and W are finite dimensional vector spaces. Assume $T \in \mathcal{L}(V, W)$. Prove if T is injective then Nullity(T) = 0.

6. Prove that $\text{Range}(T)^\circ = \text{Null}(T')$.