Math 4160 - Quiz 4

Name:____

1. Find eigenvalues and cooresponding eigenvectors for the following operators

   (a) $T \in \mathcal{L}(\mathcal{P}_2(\mathbb{R}))$ where $T(p) = p'$

   (b) $T \in \mathcal{L}(\mathcal{P}_2(\mathbb{R}))$ where $T(p) = xp'$

2. Let $V = U \oplus W$. Assume U and W are nonzero subspaces of V. And define $P \in \mathcal{L}(V)$ by $P(u + w) = u$ where $u \in U$ and $w \in W$. Find all eigenvalues and eigenvectors of P.

3. Let $T \in \mathcal{L}(V)$ and let $v, w \in V$ be nonzero vectors so that

$$T(u) = 3w \text{ and } T(w) = 3u.$$

   Prove that either 3 or -3 is an eigenvalue of T.

4. Let $T \in \mathcal{L}(V)$ and assume $T^n = 0$ where $n \in \mathbb{N}$.

   (a) Show $I - T$ is invertible and that

$$(I - T)^{-1} = I + T + T^2 + \cdots + T^{n-1}$$

   (b) Note $I - 2^n T$ is invertible also. what is $(I - 2^n T)^{-1}$?

   (c) What if we know that $I - T^3 = 0$. Then guess T^{-1}