Math 4160 - Quiz 3

Name:____

1. Write the definition of a linear map.

2. Prove the following are linear maps or prove they are not.

   (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (ax+by, cx+dy)$ where $a, b, c, d \in \mathbb{R}$.

   (b) Let $T : \mathbb{R}^3 \to \mathbb{R}$ be defined by $T(x, y, z) = x - y + 3z + 2$.

   (c) Let $L : \mathbb{F}^\infty \to \mathbb{F}^\infty$ be defined by $L(x_1, x_2, x_2, x_3, \ldots) = (x_2, x_3, x_4, \ldots)$.

3. Prove $\mathcal{L}(V, W)$ is a vector space.

4. Compute the Null space and range for the following.

   (a) Let $T : \mathbb{R}^3 \to \mathbb{R}$ be defined by $T(x, y, z) = x - y + 3z$.

   (b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (2x, y - 3x)$.

   (c) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(x, y, z) = (x + y, 2y - 2x)$.

5. For the linear maps defined in Quesion 4, compute Rank, Nulllity and the dimension of V, where V is the domain.

6. Prove the Null(T) is a vector space.

7. Asume $T \in \mathcal{L}(V, W)$. Complete the following:

   - T is injective if and only if
   - T is surjective if and only if
   - If T is injective and surjective we say T is a bijection. When T s a bijection we say T is an ____________________________ from V to W and we say that V and W are ____________________________.

8. State the Fundamental Theorem of Linear Algebra.

9. For the linear maps defined in Question 4 compute their matrices. Use the standard bases.

10. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $Tv = v$.

    (a) Using $B_1 = (1, 2), (1, 3)$ as the basis for the domain and $B_2 = (1, 0), (2, 2)$ as the basis for the codomain, compute the matrix for T. Call this matrix M_1.

(b) Switch the roles of B_1 and B_2, compute the matrix for T. Call this matrix M_2.

(c) Compute $M_1 M_2$ and $M_2 M_1$.

(d) What does Question 10c tell us about T?