**Math 3520 - Test 2**

**Name:**________________________________________________

1. Let $a, b, c, d \in \mathbb{Z}$ with $b \neq 0$. Show if $a|b$ and $b|c$ then $a|c$.

2. For the following pairs of numbers, find their gcd and and find a linear combination of the numbers equal to their gcd: $a = 144$ and $b = 252$ using the algorithm from class.

3. Let $(G, *, e)$ be a group prove
   If $a^2 = e$ for all $a \in G$ then G is Abelian.

4. For the group $(\mathbb{Z}, +)$ prove, using the 2-step subgroup test, that $H = \{4n : n \in \mathbb{Z}\}$ is a subgroup.

5. For the group $(Z_9^*, \cdot)$ list and compute the orders of its elements.

6. The groups $(Z_6, +)$ and $(Z_9^*, \cdot)$ are isopmorphic.

   (a) Define an isomorphism $f : Z_6 \to Z_9^*$. Define the isomorphism clearly. No need to show it is a bijection.

   (b) Verify your isomorphism works for the following:

   $$f(2 + 3) = f(2) \cdot f(3) \text{ and } f(4 + 4) = f(4) \cdot f(4)$$

7. For the following algebraic structures write the operation tables. The structure is not a group. State which property fails and how.

   $(D, \circ)$ a subset of $S_3$ where

   $$D = \{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}\}$$