Math 3520 - Test 1 Review

Be certain to **know** the quizzes and . . .

1 Preliminaries

1. Induction, definition of odd, even, sets union, intersection complement
2. Prove $8|5^{2n} - 1$ for all $n \in \mathbb{N}$.

2 Relations

3. Define relation, equivalence relation, well ordered, reflexive, symmetic, trasitive, domain, codomain, partirion of a set
4. We define the given relation from A to B by

$$R = \{(1,a),(2,a),(3,a),(4,a),(1,a),(2,a)\}$$

where $A = \{1,2,3,4\}$ and $B = \{a,b,c\}$. What are the domain and codomain?

5. We define the given relation on A by

$$R = \{(1,1),(2,1),(3,1),(4,1),(1,2),(2,2)\}$$

where $A = \{1,2,3,4\}$.

(a) What are the domain and codomain?
(b) Is R reflexive? If it is not reflexive, expand R so that it is reflexive.
(c) Is R symmetric? If it is not symmetric, expand R so that it is symmetric.
(d) Is R transitive? If it is not transitive, expand R so that it is transitive.
(e) Is R an equivalense relation?

6. We define the given relation on A by

$$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(3,2),(2,3),(a,b)(c,d)\}$$

where $A = \{1,2,3,4\}$.

(a) Assume R is an equivalence relation. Find (a, b) and (c, d).
(b) What are the equivalence classes for R.

7. We define the given relation on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 4|3a - b.$$

   (a) Prove R is reflexive. May not be.
   (b) Prove R is symmetric. May not be.
   (c) Prove R is transitive. May not be.

8. We define the given relation on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 4|3a + b.$$

   (a) Prove R is reflexive.
   (b) Prove R is symmetric.
   (c) Prove R is transitive.
   (d) What are the equivalence classes for R.

3 Functions

9. Define functiom, injective, surjective, domain, codomain, range, inverse image, inverse function, permutations

10. Define $f : \{1, 2, 3\} \to \{4, 7, 9\}$ by $f(1) = 4$, $f(2) = 4$ and $f(3) = 9$.

    (a) Is f injective, surjective or bijective? Compute
    (b) Compute $f(\{1, 2\})$, $f^{-1}(\{1, 4\})$ and $f \circ f^{-1}(\{4, 9\})$.

11. Define $f : \mathbb{Z} \to \mathbb{Z}$ by $f(n) = 2n - 1$. Is f injective, surjective or bijective? Prove or disprove.

12. Define $f : (-\infty, 0) \to [0, \infty)$ by $f(x) = x^2$.

    (a) Is f injective, surjective or bijective? Prove or disprove.
    (b) Compute $f((-2, 2))$, $f^{-1}((-2, 2))$, $f^{-1} \circ f(\{4, 9\})$ and $f \circ f^{-1}(\{4, 9\})$.

13. Define $f : (-\infty, 0] \to [0, \infty)$ by $f(x) = x^2$.

    (a) Is f injective, surjective or bijective? Prove or disprove.

(b) Compute $f((-2,2))$, $f^{-1}((-2,2))$, $f^{-1}\circ f(\{4,9\})$ and $f\circ f^{-1}(\{4,9\})$.

14. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^2$.

    (a) Is f injective, surjective or bijective? Prove or disprove.

    (b) Compute $f((-2,2))$, $f^{-1}((-2,2))$, $f^{-1}\circ f(\{4,9\})$ and $f\circ f^{-1}(\{4,9\})$.

15. Define $f : \mathbb{R} - \{2\} \to \mathbb{R} - \{1\}$ by $f(x) = \frac{x}{x-2}$. Is f injective, surjective or bijective? Prove or disprove.

16. Let $f : A \to B$ and $g : B \to C$. Prove the following.

    (a) If f and g are injective then $g \circ f$ is injective.

    (b) If f and g are surjective then $g \circ f$ is surjective.

4 Cardinality

17. Define cardinality. That is $A \sim B$ if and only if ...

18. Know $\sim$ is an equivalence relation and what that means.

19. Show $\mathbb{N} \sim 2\mathbb{N}$

20. Let $A = [0,2]$ and $B = [-1,6]$. Show $f : A \to B$ given by $f(x) = \frac{7}{2}x - 1$ is a bijection. What does this tell us about the sets A and B?

21. Show the sets have the same cardinality

    (a) $\mathbb{N}$ and $\mathbb{Z}$

    (b) $\mathbb{N}$ and $\mathbb{Q}$

    (c) $\{a, b, c\}$ and $\{1, 2, 7\}$

    (d) $\mathbb{N} \times \{1, 2, 3, 4\}$ and $\mathbb{N}$

    (e) $[0, 1)$ and $(2, 3]$

    (f) $(0, 1)$ and $\mathbb{R}$

22. Show the sets **do not** have the same cardinality

    (a) $\mathbb{N}$ and $\mathbb{R}$

23. Prove the irrationals are uncountable.

5 Permutations

24. Find all bijective functions from $\{1,2,3\}$ to $\{1,2,3\}$. How many functions did you come up with?

25. List all elements of the set $\mathcal{S}_3$. How many elements are in the set $\mathcal{S}_6$?

26. For the following permutaions:
$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ and $\sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$
compute:

    (a) $\sigma_1 \circ \sigma_1$

    (b) $\sigma_1 \circ \sigma_2 \circ \sigma_2$

    (c) σ_1^3

    (d) σ_1^{-1}