Math 3520 - Test 1

Name:_____

1. Prove $7|4^{3n} - 1$ for all $n \in \mathbb{N}$.
2. We define the given relation on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 3|a^2 - b^2.$$

   (a) Prove R is reflexive.
   (b) Prove R is symmetric.
   (c) What are the equivalence classes for R.

3. Prove: If f and g are surjective then $g \circ f$ is surjective.
4. Prove $(3, 1)$ and $(-1, 4)$ have the same cardinality.
5. Prove **one** of the following:

   (a) $\mathbb{N} \sim 3\mathbb{N}$ have the same cardinality.
   (b) Prove $\mathbb{R}$ is uncountable.

6. For the following permutaions:
   $\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 2 & 3 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ and
   $\sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ compute:

   (a) $\sigma_1 \circ \sigma_2 \circ \sigma_3$
   (b) σ_1^3
   (c) σ_1^{-1}