Math 3520 - Quiz 4

Name:____

1. Prove each of the following is or is not an equivalence relation

   (a) $\mathcal{R}$ is defined on $\mathbb{Z}$ as follows
   $$a\mathcal{R}b \Leftrightarrow a - b = 2$$
   (b) $\mathcal{R}$ is defined on $\mathbb{N}$ as follows
   $$a\mathcal{R}b \Leftrightarrow a = b$$
   (c) $\mathcal{R}$ is defined on $\mathbb{Z}$ as follows
   $$a\mathcal{R}b \Leftrightarrow 2|a - b$$
   (d) $\mathcal{R}$ is defined on $\mathbb{Z}$ as follows
   $$a\mathcal{R}b \Leftrightarrow |a - b| = 2$$

2. What are the equivalence classes for the relation defined below? We define the relation $\mathcal{R}$ on the set $A = \{1, 2, 3, 4\}$ by
   $$\mathcal{R} = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)\}$$

3. Compute the modular arithmatic problems below. Make sure your answers are in the range $\{0, 1, 2, 3, \ldots, n-1\}$ when computing $\mod n$.

   (a) $3^3 \mod 6$
   (b) $5^7 \mod 6$
   (c) $2^{100} \mod 7$
   (d) $100^2 \mod 7$

4. Find all x so that

   (a) $2 \cdot x = 3 \mod 6$
   (b) $2 \cdot x = 4 \mod 6$
   (c) $2 \cdot x = 3 \mod 7$
   (d) $2 \cdot x = 4 \mod 7$

5. From Problem 4, conjecture when does the equation
   $$2 \cdot x = b \mod n$$
   have a unique solution? What conditions should be placed on n?

6. Prove modular addition is well defined.