Math 3520 - Quiz 3

Name:_____

For this assignment prove or disprove the following. Type out your answers with correct mathematics and with correct English.

1. Which of the following sets are well ordered?

   (a) $S = \{x \in \mathbb{Q} | x \geq -10\}$

   (b) $S = \{-2, -1, 0, 1, 2\}$

   (c) $S = \{x \in \mathbb{Q} | -1 \leq x \leq 1\}$

   (d) $S = \{p | p \text{ is a prime}\} = \{2, 3, 5, 7, 11, \ldots\}$

   (e) $S = \{\frac{1}{n} | n \in \mathbb{N}\}$

2. Prove: $1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \in \mathbb{N}$.

3. Prove: $n^2 < 2^n$ for all $n \in \mathbb{N}$ where $n \geq 5$.

4. Prove: $8 | (5^{2n} - 1)$ for all $n \in \mathbb{N}$.

5. Prove: Define a sequence $a_1 = 1$ and $a_n = 2a_{n-1}$ for all $n \in \mathbb{N}$. Guess a formula for this sequence and prove this sequence satisfies that formula.

6. Prove: Define a sequence $a_1 = 1$ and $a_n = \sqrt{2 + a_{n-1}}$ for all $n \in \mathbb{N}$. Prove this formula is monotone.