MATH 2320 Practice Test 3

1. **Root Test**
   (a) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k^2+1}{2k^2-1}\right)^k$
   (b) $\displaystyle\sum_{k=1}^{\infty}\left(\frac{3k^2+1}{2k^2-1}\right)^k$
   (c) $\displaystyle\sum_{k=1}^{\infty}\left(1-\frac{1}{k}\right)^{k^2}$
   (d) $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{2}{k}\right)^{k^2}$
2. **Divergence Test**
   (a) $\displaystyle\sum_{k=1}^{\infty}\frac{k^2+k}{2k^2-1}$
   (b) $\displaystyle\sum_{k=1}^{\infty}\left(1+\frac{1}{2k-1}\right)^k$
   (c) $\displaystyle\sum_{k=1}^{\infty}\frac{2^k+1}{2k^2+4}$
3. **Comparison Test**
   (a) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^3+1}$
   (b) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^2+1}$
   (c) $\displaystyle\sum_{k=3}^{\infty}\frac{1}{k-1}$
   (d) $\displaystyle\sum_{k=2}^{\infty}\frac{1}{\sqrt{k}-1}$
4. **Limit Comparison Test**

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{3k^2+1}{2k^5-1}$

(b) $\displaystyle\sum_{k=1}^{\infty} \sqrt{\frac{3k^2+1}{2k^3-1}}$

(c) $\displaystyle\sum_{k=1}^{\infty} \sqrt{\frac{3k^2+1}{2k^4-1}}$

(d) $\displaystyle\sum_{k=1}^{\infty} \sqrt{\frac{3k^2+1}{2k^5-1}}$

(e) $\displaystyle\sum_{k=1}^{\infty} \frac{\sin(\frac{1}{k})}{\sqrt{k}}$. Hint: try comparing to $\sum b_k = \sum \frac{1}{k^{3/2}}$.

5. **Alternating Series Test**

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}(-1)^k$

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{k+1}(-1)^k$

(c) $\displaystyle\sum_{k=1}^{\infty} \left(1-\frac{1}{k}\right)^k (-1)^k$

6. **Approximating Functions with Polynomials**

(a) Let $f(x) = \ln(2x-3) + 2x$ find a polynomial of degree 3 that approximates $f(x)$ near the point $a = 2$. Use the polynomial to approximate $f(3)$.

(b) Let $f(x) = x^5$ find a polynomial of degree 3 that approximates $f(x)$ near the point $a = 2$. Use the polynomial to approximate $f(3)$.

7. **Power Series** Find the radius of convergence and interval of convergence for the following power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}x^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}x^n$

(c) $\sum_{n=1}^{\infty} \frac{n^2+1}{n+1} x^n$

(d) $\sum_{n=1}^{\infty} \frac{1}{2^n} x^n$

(e) $\sum_{n=1}^{\infty} \frac{1}{n!} x^n$

(f) $\sum_{n=1}^{\infty} \frac{3^n}{4^n+1} x^n$

(g) $\sum_{n=1}^{\infty} \frac{1}{2n} x^{2n}$

8. **Taylor Series** Find the Taylor series for the following functions centered at a

   (a) $f(x) = e^{2x}$ where $a = 0$

   (b) $f(x) = e^{2x}$ where $a = 1$

   (c) $f(x) = e^x$ where $a = 0$

   (d) $f(x) = \sin(x)$ where $a = 0$

   (e) $f(x) = \cos(x)$ where $a = 0$

   (f) $f(x) = \frac{1}{1-x}$ where $a = 0$

9. **Taylor Series** Find the Taylor series using known series.

   (a) $f(x) = e^{2x}$ where $a = 0$

   (b) $f(x) = x^2 e^x$ where $a = 0$

   (c) $f(x) = \frac{e^x-1}{x}$ where $a = 0$

   (d) $f(x) = \frac{1}{1+x}$ where $a = 0$

   (e) $f(x) = \frac{1}{1+x^2}$ where $a = 0$

   (f) $f(x) = \arctan(x)$ where $a = 0$

10. **Conic Sections** Graph the following

    (a) $x^2 + y^2 = 1$

    (b) $x^2 - y^2 = 1$

    (c) $-x^2 + y^2 = 1$

(d) $-x^2 - y^2 = 1$

(e) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

(f) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(g) $-x^2 + y = 1$

(h) $-x^2 - y = 1$

(i) $x - y^2 = 1$

(j) $y = x^2$