Math 7500 - Test 1

Name:_____

1. Name the five regular polyhedra (and describe) Prove that these are the only possible regular polyhedra.

2. Find all integer solutions to

$$104x + 168y = 88$$

3. Find all solutions to

$$\begin{aligned} x &\equiv 3 \mod 7 \\ x &\equiv 2 \mod 5 \\ x &\equiv 1 \mod 2 \end{aligned}$$

4. Continued fractions.

   Turn the continued fractions into a real number representation.

$$1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{2}}}}$$

$$1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cdots}}}}$$

5. You all remember the Basel problem. Present this. Here are a few of the steps.

   (a) What is the MacLaurin series for the function $\sin(x)$?

   (b) What is the MacLaurin series for the function $\frac{\sin(x)}{x}$?

   (c) Find a polynomial with roots $\pm\pi, \pm 2\pi, \pm 3\pi, \ldots$ with constant term 1.

6. Can be solved with a diophantine equation.
The Monkey and the Coconuts

Five men and a monkey were shipwrecked on a desert island, and they spent the first day gathering coconuts for food. Piled them all up together and then went to sleep for the night.

But when they were all asleep one man woke up, and he thought there might be a row about dividing the coconuts in the morning, so he decided to take his share. So he divided the coconuts into five piles. He had one coconut left over, and gave it to the monkey, and he hid his pile and put the rest back together.

By and by, the next man woke up and did the same thing. And he had one left over and he gave it to the monkey. And all five of the men did the same thing, one after the other; each one taking the fifth of the coconuts in the pile when he woke up, and each one having one left over for the monkey. And in the morning they divided what coconuts were left, and they came out in five equal shares. Of course each one must have known that there were coconuts missing; but each one was guilty as the others, so they didnt say anything. How many coconuts were there in the beginning?