**Math 3520 - Test 1 Review**

Be certain to **know** the quizzes and . . .

# 1 Preliminaries

1. Induction, definition of odd, even, sets union, intersection complement

2. Prove $8|5^{2n} - 1$ for all $n \in \mathbb{N}$.

3. Prove $n! > 2^n$ for all $n \in \mathbb{N}$ with $n > 3$.

4. Define the sequence

$$a_1 = 1 \text{ and } a_n = \sqrt{a_{n-1} + 3}$$

   (a) Prove $a_n$ is increasing. That is, prove $a_n \le a_{n+1}$ for all $n \in \mathbb{N}$.

   (b) Prove $a_n$ is bouinded above by 6. That is, prove $a_n \le 6$ for all $n \in \mathbb{N}$.

# 2 Relations

5. Define relation, equivalence relation, well ordered, reflexive, symmetic, trasitive, domain, codomain, partirion of a set

6. Which of the following are well ordered?

   - $A = \{1, 2, 3\}$
   - $B = \{-n|n \in \mathbb{N}\}$
   - $C = \{p|p \in \mathbb{Z}$ and p is prime$\}$
   - $\mathbb{Z}$
   - $E = \{n|n \in \mathbb{N}$ and $3|n\}$
   - $F = \{n|n \in \mathbb{Z}$ and $3|n\}$
   - $G = \{n|n \in \mathbb{N}$ and $3 \le n \le 45\}$
   - $H = \{n|n \in \mathbb{Q}$ and $3 \le n \le 45\}$
   - $I = \{1 - \frac{1}{n}|n \in \mathbb{N}\}$

7. Prove: If A is well ordered and $B \subseteq A$ then B is well ordered.

8. We define the given relation from $A$ to $B$ by

$$R = \{(1,a),(2,a),(3,a),(4,a),(1,a),(2,a)\}$$

where $A = \{1,2,3,4\}$ and $B = \{a,b,c\}$. What are the domain and codomain?

9. We define the given relation on $A$ by

$$R = \{(1,1),(2,1),(3,1),(4,1),(1,2),(2,2)\}$$

where $A = \{1,2,3,4\}$.

   (a) What are the domain and codomain?
   (b) Is R reflexive? If it is not reflexive, expand $R$ so that it is reflexive.
   (c) Is R symmetric? If it is not symmetric, expand $R$ so that it is symmetric.
   (d) Is R transitive? If it is not transitive, expand $R$ so that it is transitive.
   (e) Is R an equivalense relation?

10. We define the given relation on $A$ by

$$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(3,2),(2,3),(a,b)(c,d)\}$$

where $A = \{1,2,3,4\}$.

   (a) Assume R is an equivalence relation. Find $(a,b)$ and $(c,d)$.
   (b) What are the equivalence classes for $R$.

11. We define the given two relations below on $\mathbb{Z}$. Answer the questions for each of them. One relation is an equivalence relation the other is not an equivalence relation

   - $aRb \Leftrightarrow 4|3a-b$.
   - $aRb \Leftrightarrow 4|3a+b$.

   (a) Prove $R$ is reflexive.
   (b) Prove $R$ is symmetric.
   (c) Prove $R$ is transitive.
   (d) What are the equivalence classes for $R$.

## 3 Functions

12. Define functiom, injective, surjective, domain, codomain, range, inverse image, inverse function, permutations

13. Define $f : \{1,2,3\} \to \{4,7,9\}$ by $f(1) = 4$, $f(2) = 4$ and $f(3) = 9$.

    (a) Is $f$ injective, surjective or bijective? Compute

    (b) Compute $f(\{1,2\})$, $f^{-1}(\{1,4\})$ and $f \circ f^{-1}(\{4,9\})$.

14. Define $f : \mathbb{Z} \to \mathbb{Z}$ by $f(n) = 2n - 1$. Is $f$ injective, surjective or bijective? Prove or disprove.

15. Define $f : (-\infty, 0) \to [0, \infty)$ by $f(x) = x^2$.

    (a) Is $f$ injective, surjective or bijective? Prove or disprove.

    (b) Compute $f((-2,2))$, $f^{-1}((-2,2))$, $f^{-1} \circ f(\{4,9\})$ and $f \circ f^{-1}(\{4,9\})$.

16. Define $f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \setminus \{1\}$ by $f(x) = \frac{x}{x-2}$. Is $f$ injective, surjective or bijective? Prove or disprove.

17. Let $f : A \to B$ and $g : B \to C$. Prove the following.

    (a) If $f$ and $g$ are injective then $g \circ f$ is injective.

    (b) If $f$ and $g$ are surjective then $g \circ f$ is surjective.

    (c) If $g \circ f$ are injective then $f$ is injective.

    (d) If $g \circ f$ are surjective then $g$ is surjective.

18. Find all bijective functions from $\{1,2,3\}$ to $\{1,2,3\}$. How many functions did you come up with?

19. List all elements of the set $\mathcal{S}_3$. How many elements are in the set $\mathcal{S}_6$?

20. For the following permutaions:
    $\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ and $\sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$
    compute:

    (a) $\sigma_1 \circ \sigma_1$

    (b) $\sigma_1 \circ \sigma_2 \circ \sigma_2$

    (c) $\sigma_1^3$

    (d) $\sigma_1^{-1}$

## 4 Cardinality

21. Define
    - cardinality. That is $A \sim B$ if and only if $\ldots$
    - Countable, Uncountable and finite
22. Know $\sim$ is an equivalence relation and what that means.
23. Show $\mathbb{N} \sim 2\mathbb{N}$
24. Show $\mathbb{N} \sim \mathbb{Z}$
25. Let $A = [0, 2]$ and $B = [-1, 6]$. Show $f : A \to B$ given by $f(x) = \frac{7}{2}x - 1$ is a bijection. What does this tell uas about the sets A and B?
26. Prove $\mathbb{N} \sim \mathbb{Z}$
27. Prove $\mathbb{N} \sim \mathbb{Q}$
28. Prove $\mathbb{N} \not\sim \mathbb{R}$
29. Prove $\mathbb{R} \setminus \mathbb{Q}$ is uncountable.
30. Prove if A and B are countably infinite then $A \cup B$ is countablbly infinite.
31. Let $A \subseteq \mathbb{N}$. Show A is either finite or A is countably infinite.