Math 3520 - Test 1

Name:_____

1. Prove $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for all $n \in \mathbb{N}$.

2. Define the relation on $\mathbb{Z}$ by

$$x \mathcal{R} y \Leftrightarrow 3 \mid 5x + y$$

Prove $\mathcal{R}$ is transitive.

3. Define $f : (0, \infty) \to (0, \infty)$ by $f(x) = \frac{1}{x}$. Is f injective, surjective or bijective? Prove or disprove.

4. Let $f : A \to B$ and $g : B \to C$. Prove: If f and g are surjective then $g \circ f$ is surjective.

5. For the following permutaions:
$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \text{ and } \sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$$
compute:

(a) σ_1^3

(b) σ_1^{-1}

6. Prove $(1, 2) \sim (9, 13)$

7. Prove the set of irrationals is uncountable.