Math 3160 - Test 2 Review

Be certain to **know** the quizzes and . . .

1 Vector Spaces and Subspaces

1. Let $V = \mathbb{R}^3$ equipped with usual vector addition and scalar multiplication. Prove V is a vector space. That is, prove all 10 Axioms.

2. Let $V = \mathbb{R}^2$. And define the two operations

   $\oplus$: $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1)$

   $\odot$: $k \odot (x_1, y_1) = (kx_1, ky_1)$

   (a) Compute $(0, 4) \oplus (-2, 3)$ and compute $2 \odot (1, 1)$.

   (b) Show $\mathbf{0} \neq (0, 0)$.

   (c) Show $\mathbf{0} = (-1, -1)$.

   (d) Prove Axiom 5. That is, for each $\mathbf{v}$ find $-\mathbf{v}$ so that

   $$\mathbf{v} \oplus -\mathbf{v} = \mathbf{0}.$$

   (e) V does satisfy some of the vector space axioms, but not all of the axioms. Find two axioms that fail.

3. State the two step subspace test.

4. Let $W = \{(a, b, c) \in \mathbb{R}^3 : \text{ where } a + b + c = 1\}$.

   (a) Use the two step subspace test to show $(W, +, \cdot)$ is a subspace.

   (b) What geometric shape is W? Hint I gave it in standard form.

   (c) Give me the parametric for the geometric object defined in the set W.

5. Let $W = \{(x, y, z) \in \mathbb{R}^3 : \text{ where } x - 3y - z = 0\}$.

   (a) Use the two step subspace test to show $(W, +, \cdot)$ is a subspace.

   (b) What geometric shape is W? Hint I gave it in standard form.

   (c) Give me the parametric for the geometric object defined in the set W.

2 Linear Independence

6. Let $S = \{(1,2,1),(0,1,2),(0,-1,0)\}$.

   (a) Is S linearlly independent? (There is an easy test for this problem).

   (b) Is $(2,2,2) \in \text{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(2,2,2)$?

   (c) Does S span $\mathbb{R}^3$?

7. Let $S = \{x, x+2, x^3 - x - 1, x^3\}$ be a set in P_3.

   (a) Is S linearlly independent?

   (b) Is $x^3 + x^2 + x + 1 \in \text{Span}(S)$? If yes what is a linear combination of the polynomials in S that equals $x^3 + x^2 + x + 1$?

   (c) Is $4x^3 - 2x \in \text{Span}(S)$? If yes what is a linear combination of the polynomials in S that equals $4x^3 - 2x$?

   (d) Does S span P_3?

3 Span, Basis

8. Let $B = \{(1,2,1),(0,1,2),(0,-1,0)\}$.

   (a) Is B a basis for $\mathbb{R}^3$

   (b) Write the vector $(1,0,-1)$ relative to the basis B.

   (c) Write the vector (a,b,c) relative to the basis B.

   (d) Find the change of basis matrix from the standard basis to the basis B. (we called it $P_{\text{STANDARD}\to B}$ in class).

9. For the following system of linear equations.

$$\begin{array}{rrrrrl} 2x_1 & -2x_2 & +4x_3 & & -6x_5 & = 2 \\ & & x_3 & +6x_4 & & = 0 \end{array}$$

   (a) Find the solution set.

   (b) Find a basis for the solution set.

   (c) What is the dimension of that solution set?

10. For the following subspace of P_3

$$W = \{a + bx + cx^2 + dx^3 : a = -c \text{ and } b = c + d\}$$

   (a) Find a basis for W.
   (b) What is the dimension of that solution set?

4 Change of Basis Matrix

11. Let $B = \{(1, 0), (0, 1)\}$, $B_1 = \{(-1, 1), (2, 3)\}$ and $B_2 = \{(1, -1), (1, 1)\}$.
   (a) Find the change of basis matrices for $P_{B_1 \to B_2}$ and $P_{B_1 \to B_2}$.
   (b) Find the coordinates of the point $(4, 6)$ (given in the standard basis) relative to the bases B_1 and B_2.
   (c) Find the change of basis matrices for $P_{B \to B_2}$ and $P_{B_2 \to B}$.
   (d) Find the coordinates of the point $(2, -4)$ (given in the standard basis) relative to the bases B and B_2. Graph this point the two separate coordinate axes B and B_2.

5 Row Space, Column Space & Null space

12. Quiz 7

6 Basic Transformations

13. Write the matrix for the following transformations described below.
   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 45° counter-clockwise.
   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis.
   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the x-axis is contracted by half and the y-axis is dilated by 2.
   (d) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 30° counter-clockwise and then reflected about the x-axis.
   (e) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the x-axis and then rotated by 30° counter-clockwise.
   (f) $T : \mathbb{R}^3 \to \mathbb{R}^3$ where the x-axis is contracted by half and the z-axis is dilated by 2.

7 Eigenvalues, Eigenvectors and Diagonalization

14. For the following matrices find the charactersitic equation, the eigenvalues and their cooresponding eigen vectors.

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$

and

$$E = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix},$$

15. for the above matrices, determine if they are diagonalizable. State why or why not. And if it is diagonalizable, diagonaalize it. That is, find P and D.

16. Diagonalize the matrix below.

$$\begin{bmatrix} 4 & 0 & -1 & -1 \\ 0 & 3 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$