Math 3160 - Test 2

Name:____

1. Let $V = \mathbb{R}^2$. And define the two operations

   $\oplus$: $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 - 1)$

   $\odot$: $k \odot (x_1, y_1) = (kx_1, y_1)$

   (a) Prove (or disprove) the V is closed under vector addition (Axiom 1).

   (b) Note V has a zero vector. What is the zero vector for V

2. Let $W = \{(a, b, c) \in \mathbb{R}^3 : \text{ where } a = 0\}$. Use the two step subspace test to show $(W, +, \cdot)$ is a subspace.

3. Let $S = \{x+1, 2x^2 - 3x - 3, x^2\}$ be a set in P_2.

   (a) Is S linearlly independent?

   (b) Is $-2x \in \text{Span}(S)$? If yes what is a linear combination of the polynomials in S that equals $-2x$?

   (c) Does S span P_2?

4. Let $B = \{(1,0,1), (0,1,2), (1,-1,0)\}$.

   (a) Is B a basis for $\mathbb{R}^3$

   (b) Write the vector $(1,0,-1)$ relative to the basis B.

5. Let $B_1 = \{(1,2), (-3,1)\}$, $B_2 = \{(-1,1), (2,3)\}$.

   (a) Find the change of basis matrices for $P_{B_1 \to B_2}$ and $P_{B_1 \to B_2}$.

   (b) Find the coordinates of the point $(4,6)$ (given in the standard basis) relative to the bases B_1 and B_2. Here you may need $P_{\text{STANDARD} \to B_1}$.

6. The linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is given by the formula
$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_3 \\ x_2 \\ x_4 \end{bmatrix}.$$

(a) Find the matrix, A, to represent the linear trasformation T.

(b) Compute the basis for the Range of T.

(c) Find a basis for the null space of A, NULL(A).

(d) Find the rank and nullity.

(e) What is the dimension of the domain of T and the codomain of T? Compare Rank, Nullity and the dimension of the Domain.

7. Write the matrix for the following transformations described below.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is rotated by 60° counter-clockwise.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ where the plane is reflected about the y-axis and then rotated by 60° counter-clockwise.

   (c) $T : \mathbb{R}^3 \to \mathbb{R}^3$ where the y-axis is contracted by a factor of $1/3$ and the z-axis is dilated by 3.

8. Diagonalize the matrix below.

$$\begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$