**Math 3160 - Test 1 Review**

Be certain to **know** the quizzes and . . .

# 1 Systems of Linear Equations

1. Solve the following systems of linear equations using row reduction.

(a) $$\begin{cases} x_1 & -2x_2 & & & -6x_5 & = 0 \\ & x_2 & +x_3 & +6x_4 & & = 5 \\ & 2x_2 & & +6x_4 & +x_5 & = 4 \\ & x_2 & -x_3 & & +x_5 & = -1 \end{cases}$$

(b) $$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & & x_3 & = 0 \\ x_1 & +x_2 & +2x_3 & = 0 \end{cases}$$

(c) $$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ -x_1 & -x_2 & +3x_3 & = 2 \\ x_1 & -3x_2 & +7x_3 & = 2 \end{cases}$$

2. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

(a) $$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$$

(b) $$\begin{cases} 2x_1 & -2y & = 2 \\ -x_1 & -3y & = 2 \end{cases}$$

# 2 Matrices, Determinants and Cramer's Rule

3. Let $A = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 4 & 1 \\ 0 & 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 0 & 0 \\ 2 & -2 & 1 & 2 \\ 2 & -2 & 0 & 3 \\ 2 & -2 & 5 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

and $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find the determinant of the matices A, B, C and D

(b) Compute $D^3$ and $D^{-1}$.

(c) Compute $C^TC$. What kind of matrix is $C^TC$?

4. Solve the following equations for $X$ assuming any matrix has an inverse. Let A, B, C, X be $nxn$ matrices and let $\mathbf{u}$ be an $nx1$ vector.

    (a) $AX = BX - A$

    (b) $AX = 2X - A$

    (c) $A\mathbf{u} = 2\mathbf{u}$

5. Assume det(A) =11.

    (a) Switch R1 and R2 in the matrix A to get matrix B, what is det(B)?

    (b) Replace R1 with R1 - 4R2 to get matrix B, what is det(B)?

    (c) Mutiply R1 by 4 to get matrix B, what is det(B)?

6. Solve the following using Cramer's rules.

    (a) $\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 2 \\ & -3x_2 & +7x_3 & = 2 \end{cases}$

    (b) $\begin{cases} 2x_1 & -2y & = 2 \\ -x_1 & -3y & = 2 \end{cases}$

## 3 Vectors

7.

8. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$. and let $P(1, 1, 1)$ and $Q(0, -4, 0)$ be two points in $\mathbb{R}^3$.

    (a) Find a vector that is parallel to $\mathbf{v}$ and unit.

    (b) Compute $\|2\mathbf{v} - \mathbf{w}\|$.

    (c) Compute the angle between $\mathbf{v}$ and $\mathbf{w}$.

    (d) Find the equation of a plane containing $P$ and with normal vector $\mathbf{v}$.

(e) Compute the distance from the point $Q$ and the plane in problem 8d.

(f) Compute the parallel component of $\mathbf{v}$ along $\mathbf{w}$ (we called this $\mathbf{w}_1 = \text{PROJ}_{\mathbf{w}}\mathbf{v}$) and compute the orthogonal components (we called this $\mathbf{w}_2$ in class).

9. Find the vector and parametric equations for the line (in $\mathbb{R}^3$) so that

   (a) the line contains the point $P(0, 1, 2)$ and is perpendicular to the vector $(1, 2, 3)$.

   (b) the line contains the point $P(-5, 1, 3)$ and is parallel to the vector $(1, 2, 3)$.

10. Find the vector and parametric equations for the plane (in $\mathbb{R}^3$) so that

    (a) the plane contains the point $P(2, 2, 2)$ and is parallel to the vectors $(1, 2, 3)$ and $(1, -1, 0)$.

    (b) the plane contains the origin and is perpendicular to the vector $(1, 2, 3)$.

11. Find the standard equation of each of the planes in Problem 10. The cross peoduct maybe helpful here.

12. Let $\mathbf{v} = (1, 3, 4)$ and $\mathbf{w} = (1, -1, 0)$ be vectors in $\mathbb{R}^3$.

    (a) Compute $\mathbf{v} \times \mathbf{w}$.

    (b) Compute $\mathbf{w} \times \mathbf{v}$.

    (c) Compute $\mathbf{w} \times \hat{\imath}$.

    (d) Compute $(\sin(\theta), \cos(\theta), 1) \times (\cos(\theta), -\sin(\theta), 0)$.

    (e) What is the area contained within the parallelogram formed by the vectors $\mathbf{v}$ and $\mathbf{w}$.

    (f) What is the area contained within the triangle defined by the three vertices $P(1, 0, 1)$, $P(-2, 0, 0)$ and the origin?

    (g) what is the volume of the parallelpiped formed by the three vectors $\mathbf{v}$, $\mathbf{i}$ and $\mathbf{w}$.