Math 3160 - Test 1

Name:_____

No calculators and show all work.

1. Solve the following systems of linear equations using row reduction.

$$\begin{cases} x_1 - 2x_2 + 2x_3 - 6x_5 = 3 \\ 4x_4 = 6 \\ x_4 - x_5 = 0 \\ 2x_2 - 2x_3 + 6x_5 = 10 \end{cases}$$

2. Solve the following systems of linear equations using row reduction.

$$\begin{cases} x_1 & +2x_2 & +3x_3 & = 4 \\ x_1 & +x_2 & +x_3 & = 2 \\ -x_1 & & -x_3 & = 4 \end{cases}$$

3. Solve the following systems of linear equations by setting up problem as a matrix problem and by finding an inverse matrix.

$$\begin{cases} x_1 & +2x_2 & +4x_3 & = 0 \\ x_1 & & -x_3 & = 2 \\ x_1 & +x_2 & +x_3 & = -3 \end{cases}$$

4. Solve the following using Cramer's rules.

$$\begin{cases} 2x_1 & -2x_2 & +4x_3 & = 2 \\ & -x_2 & +3x_3 & = 0 \\ & -3x_2 & & = 2 \end{cases}$$

5. Let T be the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ where

$$T(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} x - 3y \\ z + 2x \\ y + x \\ 4z \end{bmatrix}.$$

(a) Write the matrix, A, for the transformation T.

(b) Compute $T(\begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix})$.

(c) Compute $T(\begin{bmatrix} a \\ b \\ c \end{bmatrix}) + T(\begin{bmatrix} d \\ e \\ f \end{bmatrix})$

(d) Compute $T(\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix})$

6. Find the vector and parametric equations

   (a) for the line (in $\mathbb{R}^3$) so that the line contains the point $P(0, 1, 2)$ and is perpendicular to the vector $(1, 2, 3)$.

   (b) for the plane (in $\mathbb{R}^3$) so that the plane contains the point $P(-5, 1, 3)$ and is perpendicular to the vector $(1, 2, 3)$.

7. Row reduce the matrix B to REF and compute the determinant of the matrix B using the row reduction techniques.

$$B = \begin{bmatrix} 1 & 4 & 0 & -1 \\ 0 & 3 & 0 & -1 \\ 0 & 3 & 5 & -1 \\ 0 & 3 & 1 & 7 \end{bmatrix}$$

8. Let $P(1,3,4)$, $Q(1,3,4)$ and $R(1,3,4)$ be points in $\mathbb{R}^3$.

   (a) Compute the area of the triangle formed by the points P, Q and R.

   (b) What is the volume of the parallelpiped formed by the vectors $\vec{PQ}$, $\vec{PR}$ and $\hat{j}$?

   (c) What is the standard equation of the plane containing the triangle from Problem 8a?

   (d) What is the parametric (or vector) equation of the plane containing the triangle from Problem 8a?