Math 3160 - Final Review

Be certain to **know** the Test 1 Review, Test 2 Review and . . .

1 Recursively Defined Sequences

1. Let a sequence be defined by the following recursive formula

$$a_1 = 3, a_2 = 0 \text{ and } a_{n+2} = 3a_{n+1} - 2a_n$$

   (a) Compute the first five terms of the sequence.

   (b) Find a matrix A for the sequence as we did in class.

   (c) Diagonalize A. That is find D and P so that $A = PDP^{-1}$.

   (d) Compute $A^n \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ using Problem 1c.

2. Let a sequence be defined by the following recursive formula

$$a_1 = 4, a_2 = 1 \text{ and } a_{n+2} = a_{n+1} - 6a_n$$

   (a) Compute the first five terms of the sequence.

   (b) Find a matrix A for the sequence as we did in class.

   (c) Diagonalize A. That is find D and P so that $A = PDP^{-1}$.

   (d) Compute $A^n \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ using Problem 2c.

2 Cryptography - Hill Cipher

3. Which of the following can be used to encipher with the Hill cipher (as we did in class mod 26)?

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 25 & 5 \\ 3 & 11 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 3 & 3 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 3 & 1 & 0 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

4. Define the follwing matrix: $E = \begin{bmatrix} 5 & 1 \\ 3 & 6 \end{bmatrix}$

(a) For the plaintext message "WOLFMAN", find the two letter block representation using the alphabet: $A = 00, B = 01, C = 02, D = 03, \ldots$

(b) Use the matrix E to encipher the plaintmessage.

(c) Find the deciphering matrix D.

(d) Decipher the ciphertext: JBCMVPTKGD.

3 Markov Chain

5. For the following defined matrices which are stochastic. $A = \begin{bmatrix} .1 & .9 \\ .1 & .9 \end{bmatrix}$, $B = \begin{bmatrix} .9 & 0 \\ .1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0.25 & -1.0 \\ 0.75 & 2.0 \end{bmatrix}$ and $D = \begin{bmatrix} .1 & .2 & .3 & .4 \\ .0 & .4 & .1 & .5 \\ .0 & .4 & .6 & 0 \\ .9 & 0 & 0 & .1 \end{bmatrix}$

6. For the following defined markov processes which are regular. $A = \begin{bmatrix} .1 & .9 \\ .1 & .9 \end{bmatrix}$, $B = \begin{bmatrix} .9 & 0 \\ .1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0.25 & 1.0 \\ 0.75 & 0.0 \end{bmatrix}$

7. Find the steady states for the following processes.

   (a) For the transition matrix $A = \begin{bmatrix} .1 & .9 \\ .1 & .9 \end{bmatrix}$. You may be able to guess the answer for this one (but do the work anyway).

   (b) A bike rental company has three locations. A renter can pick up a bike at any location and drop that bike off at any location.

   - A bike picked up at location A has 20% probability of being dropped off at location B and has a 10% probability of being dropped off at location C.
   - A bike picked up at location B has 30% probability of being dropped off at location B and has a 0% probability of being dropped off at location C.
   - A bike picked up at location C has 10% probability of being dropped off at location B and has a 40% probability of being dropped off at location C.