Math 3160 - Quiz 7

Name:_____

1. Let W be the plane $x - 2y + z = 0$ in $\mathbb{R}^3$.

   (a) Find the parametric equation for the plane.

   (b) Find a basis for W.

   (c) Compute the solution set to the linear system $x - 2y + z = 0$ in $\mathbb{R}^3$.

2. Let W be the hyperplane $x_1 - 2x_2 + x_3 + 6x_4 = 0$ in $\mathbb{R}^4$.

   (a) Find the parametric equation for the hyperplane.

   (b) Find a basis for W.

   (c) Compute the solution set to the linear system $x_1 - 2x_2 + x_3 + 6x_4 = 0$ in $\mathbb{R}^4$.

3. Let $A = \begin{bmatrix} -1 & 2 & 0 & 3 & 0 \\ 2 & 1 & 1 & -1 & 1 \\ 1 & 3 & 1 & 2 & 1 \end{bmatrix}$.

   (a) Find a basis for the Column Space of A, $\text{COL}(A)$, and the row space of A, $\text{ROW}(A)$.

   (b) Compute the dimension of $\text{COL}(A)$ and $\text{ROW}(A)$.

   (c) Find a basis for thenull space of A, $\text{NULL}(A)$.

   (d) Compute the dimension of $\text{NULL}(A)$.

4. The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^5$ is given by the formula
$$T(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} x - y \\ x \\ x \\ y + z \\ 0 \end{bmatrix}.$$

   (a) Find the matrix, A, to represent the linear trasformation T.

   (b) Compute the basis for the Range of T, which is the Column Space of A.

   (c) Find a basis for the null space of A, $\text{NULL}(A)$.

(d) Compute the dimension of COL(A) and NULL(A). The dimension of the range of T is called the rank of T and the dimension of the null space is called the nullity.

(e) What is the dimension of the domain of T and the codomain of T? Compare Rank, Nullity and the dimension of the Domain.

5. The linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is given by the formula

$$T(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \begin{bmatrix} x + y \\ y + z \\ x - z \end{bmatrix}.$$

(a) Find the matrix, A, to represent the linear trasformation T.

(b) Compute the basis for the Range of T, which is the Column Space of A.

(c) Find a basis for the null space of A, NULL(A).

(d) Compute the dimension of COL(A) and NULL(A). The dimension of the range of T is called the rank of T and the dimension of the null space is called the nullity.

(e) What is the dimension of the domain of T and the codomain of T? Again, compare Rank, Nullity and the dimension of the Domain. Do you see a relation?