Math 3160 - Quiz 6

Name:____

1. Let $V = \{(a,b) \in \mathbb{R}^2 : b \neq 0\}$. And define the two operations

   $\oplus$: $(a,b) \oplus (c,d) = (ad + bc, bd)$

   $\odot$: $k \odot (a,b) = (ka, b)$

   (a) Show $(V, \oplus, \odot)$ satisfies Axiom 1.

   (b) Show $(V, \oplus, \odot)$ satisfies Axiom 2.

   (c) Does $(V, \oplus, \odot)$ satisfies Axiom 4? Why or why not?

2. Let $W = \{(a,b,c) \in \mathbb{R}^3 :$ where $a + b + c = 1\}$.

   (a) Use the two step subspace test to show $(W, +, \cdot)$ is a subspace.

   (b) What geometric shape is W? Hint I gave it in standard form.

   (c) Give me the parametric for the geometric object defined in the set W.

3. Let $S = \{(1,2,-1,1), (0,0,1,2), (1,0,-1,0)\}$.

   (a) Is S linearlly independent?

   (b) Is $(2,2,2,1) \in \text{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(2,2,2,1)$?

   (c) Is $(5,6,-8,-3) \in \text{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(5,6,-8,-3)$?

   (d) Does S span $\mathbb{R}^4$?

4. Let $S = \{(1,2,1), (0,1,2), (0,-1,0)\}$.

   (a) Is S linearlly independent? (There is an easy test for this problem).

   (b) Is $(2,2,2) \in \text{Span}(S)$? If yes what is a linear combination of the vectors in S that equals $(2,2,2)$?

   (c) Does S span $\mathbb{R}^3$?

5. Let $B = \{(1,2,1), (0,1,2), (0,-1,0)\}$.

   (a) Is B a basis for $\mathbb{R}^3$

(b) Write the vector $(1, 0, -1)$ relative to the basis B.

(c) Write the vector (a, b, c) relative to the basis B.

(d) Find the change of basis matrix from the standard basis to the basis B. (we called it $P_{\text{STANDARD}\to B}$ in class).

6. For the following system of linear equations.

$$\begin{array}{llllll} 2x_1 & -2x_2 & +4x_3 & & -6x_5 & = 2 \\ & & x_3 & +6x_4 & & = 0 \end{array}$$

(a) Find the solution set.

(b) Find a basis for the solution set.

(c) What is the dimension of that solution set?

7. For the following subspace of P_3

$$W = \{a + bx + cx^2 + dx^3 : a = -c \text{ and } b = c + d\}$$

(a) Find a basis for W.

(b) What is the dimension of that solution set?