MATH 5320 Test 1

Name:_____

1. Define the following
   (a) $f : A \to B$ is onto $\Leftrightarrow$
   (b) $\alpha = \sup(A) \Leftrightarrow$
   (c) $a_n \to a \Leftrightarrow$
2. Prove $\lim_{n\to\infty} \frac{2n+1}{3n+1} = \frac{2}{3}$ (Use the $\varepsilon - N$ definition)
3. Prove $\lim_{n\to\infty} e^n = \infty$ (Use the $M - N$ definition)
4. Prove **one** of the following
   - If $a_n \to a$ and $b_n \to b$ then $a_n b_n \to ab$
   - If (a_n) is convergent then (a_n) is bounded.
5. Do both of the following.
   (a) Show the sets $[2, 7] \sim [0, 1]$. Be certain to prove your function is bijective.
   (b) Show the sets $\mathbb{N} \sim \mathbb{Z}$. For this problem you need only define an appropriate bijection. You do not need to prove it is a bijection.
6. Show the following function is convergent (quote the appropriate theorem). And find its limit.
   $$a_1 = 4 \text{ and } a_{n+1} = \sqrt{5a_n - 6}$$
7. Prove.
   If $a_n \to 0$ and (b_n) is bounded then $a_n b_n \to 0$.