MATH 5320 Test 1.1

Name:____

1. Define the following
   (a) $f : A \to B$ is injective $\Leftrightarrow$
   (b) $\alpha = \inf(A) \Leftrightarrow$
   (c) $a_n \to \infty \Leftrightarrow$
2. Prove $(\frac{n^2+1}{3n^2+1})$ onverges. Use the $\varepsilon - N$ definition.
3. Prove $\lim_{n\to\infty} \frac{n^2}{n+1} = \infty$. Use the $M - N$ definition.
4. Prove **one** of the following.
   - If $a_n \to a$ and $b_n \to b$ then $a_n + b_n \to a + b$
   - If $a_n \to 0$ and (b_n) is bounded then $a_n b_n \to 0$.
5. Do **both** of the following.
   (a) Show the sets $(0, 1) \sim (-3, 2)$. Be certain to prove your function is bijective.
   (b) Show the sets $\mathbb{N} \not\sim \mathbb{R}$. For this problem you need only define an appropriate bijection. You do not need to prove it is a bijection.
6. Show the following function is convergent (quote the appropriate theorem). And find its limit.
$$a_1 = 1 \text{ and } a_{n+1} = \sqrt{3a_n + 4}$$
7. Prove.
   If $a_n \to a$ and $b_n \to b$ then $a_n b_n \to ab$.