

MATH 5320 Test 2: Practice

To prepare for Test 2 You should

1. PREVIOUS MATERIAL

- (1) Something from Practice Test 1 is possible.

2. SERIES

- (2) Be able to prove the divergence test.
(3) Know the proof of the geometric series test.
(4) Know the proof (at least the sketch and skeleton) of the p-series series test.
(5) Be able to show the harmonic series is divergent.
(6) Also show individual series are convergent or divergent as in class or in Quiz 4.
(7) Understand all the questions in Quiz 4.
(8) It may be useful for you to list all of the tests and propositions we have for the series.

3. LIMITS OF FUNCTIONS

- (9) Use the definition to prove $\lim_{x \rightarrow -3} x^2 = 9$.
(10) Use the definition to prove $\lim_{x \rightarrow -3} \frac{x-1}{x+7} = -1$.
(11) Use the definition to prove $\lim_{x \rightarrow 7} \frac{1}{x} = \frac{1}{7}$.
(12) Use the definition to prove $\lim_{x \rightarrow 9} \sqrt{x} = 3$.

4. CONTINUITY

- (13) Prove for $f : \mathbb{R} \rightarrow \mathbb{R}$ that $f(x) = x^2$ is continuous at $x = 2$.
(14) Prove for $f : \mathbb{R} \rightarrow \mathbb{R}$ that $f(x) = x^2$ is continuous.

5. DERIVATIVES

- (15) Prove if $f(x)$ is differentiable at $x = c$ then $f(x)$ is continuous at $x = c$.
(16) Prove if $f(x)$ and $g(x)$ are differentiable at $x = c$ then $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$.
(17) Compute the derivative
(a) $f'(x)$ for $f(x) = \sqrt{x}$
(b) $f'(x)$ for $f(x) = \frac{1}{x}$
(c) $f'(0)$ for $f(x) = \begin{cases} \frac{x^3 - x^2 \cos(x)}{|x|} & : x \neq 0 \\ 0 & : x = 0 \end{cases}$.
(18) Let $f : [0, 1] \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} \sin 1/x & : x \neq 0 \\ 0 & : x = 0 \end{cases}$.
(a) Is $f(x)$ continuous at $x = 0$? Prove your answer.
(b) Is $f(x)$ differentiable at $x = 0$? Prove your answer and compute the derivative.
(19) Let $f : [0, 1] \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} x \sin 1/x & : x \neq 0 \\ 0 & : x = 0 \end{cases}$.
(a) Is $f(x)$ continuous at $x = 0$? Prove your answer.
(b) Is $f(x)$ differentiable at $x = 0$? Prove your answer and compute the derivative.
(20) Let $f : [0, 1] \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} x^2 \sin 1/x & : x \neq 0 \\ 0 & : x = 0 \end{cases}$.
(a) Is $f(x)$ continuous at $x = 0$? Prove your answer.
(b) Is $f(x)$ differentiable at $x = 0$? Prove your answer and compute the derivative.