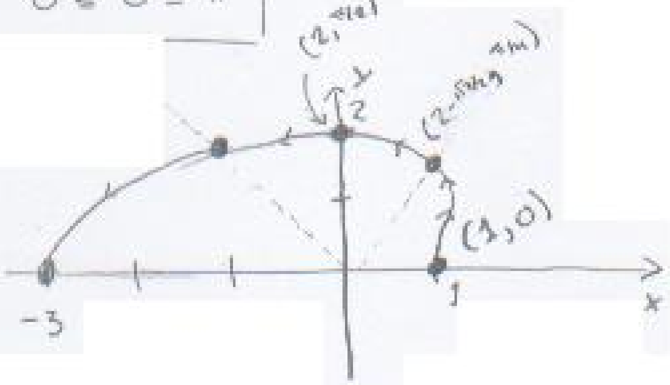
1. GRAPH ONE OF THE FOLLOWING

(a) $r = 2 - \cos(\theta)$ over $0 \le \theta \le \pi$

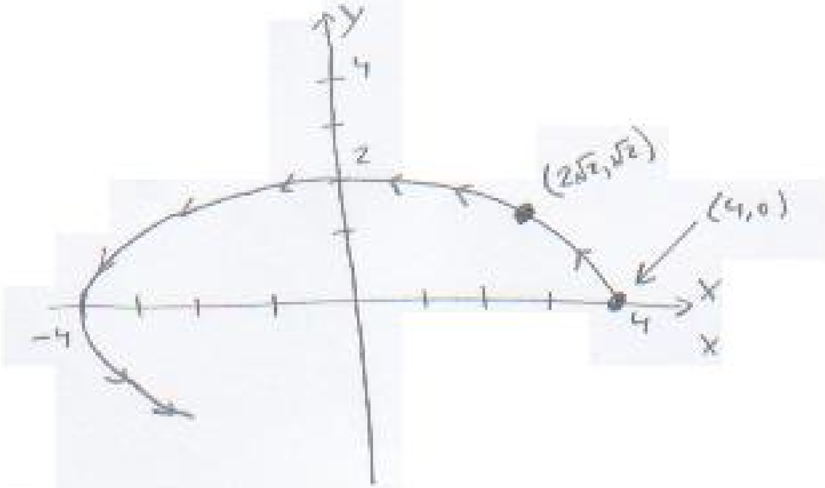
θ	r
0	1
$\pi/4$	$2 - \frac{\sqrt{2}}{2} \approx 2 - .7 = 1.3$
$\pi/2$	2
$3\pi/4$	$2 + \sqrt{2}/2 \approx 2 + .7 = 2.7$
π	3



(b) $x = 4\cos t$

$y = 2\sin t$

t	x	y
0	4	0
$\pi/4$	$2\sqrt{2}$	$\sqrt{2}$
$\pi/2$	0	2
$3\pi/4$	$-2\sqrt{2}$	$\sqrt{2}$
π	-4	0



(2) Define $A(0,2,1)$, $B(-1,3,1)$ & $C(1,-1,0)$

(a) FIND plane containing A, B & C.

$\vec{AB} = \langle -1-0, 3-2, 1-1 \rangle = \langle -1, 1, 0 \rangle$

$\vec{AC} = \langle 1-0, -1-2, 0-1 \rangle = \langle 1, -3, -1 \rangle$

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 1 & -3 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -3 & -1 \end{vmatrix} \hat{i} - \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} \hat{j} + \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} \hat{k}$$

$$= -\hat{i} - (1-0)\hat{j} + (3-1)\hat{k} = \langle -1, -1, 2 \rangle$$

So

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$-1(x - 0) - 1(y - 2) + 2(z - 1) = 0$

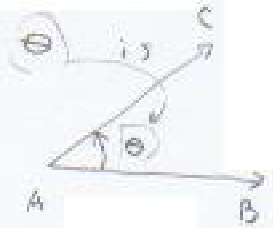
(2)(b) FIND AREA OF TRIANGLE A, B, C.

AREA $= \frac{1}{2} \| \underbrace{\overrightarrow{AB} \times \overrightarrow{BC}} \| = \frac{1}{2} \| \langle -1, -1, 2 \rangle \|$

from part (a) $\quad = \frac{1}{2} \sqrt{1+1+4} = \boxed{\sqrt{6}/2}$

(c) FIND ANY ANGLE FROM $\triangle ABC$.

$\overrightarrow{AB} \cdot \overrightarrow{AC} = \| \overrightarrow{AB} \| \, \| \overrightarrow{AC} \| \cos(\theta)$ where θ is



$\langle -1, 1, 0 \rangle \cdot \langle 1, -3, -1 \rangle = \| \langle -1, 1, 0 \rangle \| \, \| \langle 1, -3, -1 \rangle \| \cos\theta$

$-1 - 3 + 0 = \sqrt{1+1+0} \cdot \sqrt{1+9+1} \cos\theta$

$-4 = \sqrt{2} \cdot \sqrt{11}$

$\cos^{-1}\left(\frac{-4}{\sqrt{22}}\right) = \theta$

(3) $\vec{r}(t) = \langle t^2, \cos t^2 \ \sin t^2 \rangle$ COMPUTE arc length from $t=0$ to $t=\pi$

$S = \int_0^{\pi} \| \vec{r}\,'(t) \| \, dt$

arc length

$\vec{r}\,'(t) = \langle 2t, -2t \sin(t^2), 2t \cos t^2 \rangle$

$\| \vec{r}\,'(t) \| = \| \langle \qquad \rangle \|$

$= \sqrt{4t^2 + 4t^2 \sin^2 t^2 + 4t^2 \cos^2 t^2}$

$= \sqrt{4t^2 + 4t^2 (\sin^2 t^2 + \cos^2 t^2)}$

$\sqrt{4t^2 + 4t^2} = \sqrt{8}\, t = 2\sqrt{2}\, t$

So

$\int_0^{\pi} \| \vec{r}\,'(t) \| \, dt = \int_0^{\pi} 2\sqrt{2}\, t \, dt$

$\sqrt{2}\, t^2 \Big|_0^{\pi} = \boxed{\sqrt{2}(\pi^2 - 0)}$

(4.) $\vec{r}(t) = \langle 2t^3+t, t^2 \rangle$

(a) Compute & graph velocity and the acceleration at $t=-1$ & at $t=+1$

$\vec{v}(t) = \vec{r}\,'(t) = \langle 6t^2+1, 2t \rangle$

$\vec{a}(t) = \vec{r}\,''(t) = \langle 12t, 2 \rangle$

$t=-1$

$\vec{s}(-1) = \langle -3, 1 \rangle$

$\vec{v}(-1) = \langle 7, -2 \rangle$

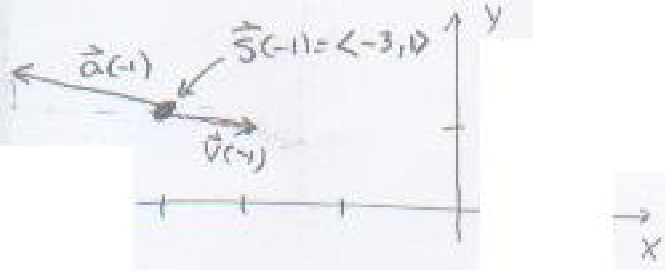
$\vec{a}(-1) = \langle -12, 2 \rangle$

$t=1$

$\vec{s}(1) = \langle 3, 1 \rangle$

$\vec{v}(1) = \langle 7, 2 \rangle$

$\vec{a}(1) = \langle 12, 2 \rangle$



(b) Compute tangent line at $t=-1$

$$\left.\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \end{aligned}\right\} \longrightarrow \boxed{\begin{aligned} x &= -3 + 7t \\ y &= 1 - 2t \end{aligned}}$$

$(x_0, y_0) = \vec{r}(-1) = \langle -3, 1 \rangle$

$\langle a, b \rangle = \vec{r}\,'(-1) = \langle 7, -2 \rangle$

(5) Prove if $\|\vec{r}(t)\|$ is CONSTANT then $\vec{r}$ is perp. to $\vec{r}'$.

proof: We will compute $\frac{d}{dt}[\|\vec{r}(t)\|^2]$ in two ways

(1) $\frac{d}{dt}\left[\|\vec{r}(t)\|^2\right] = 0$ since $\|\vec{r}(t)\|^2$ is CONSTANT.

(2) $\frac{d}{dt}[\|\vec{r}(t)\|^2] = \frac{d}{dt}[\vec{r}\cdot\vec{r}]$ Since $\|\vec{V}\|^2 = \vec{V}\cdot\vec{V}$

$= \vec{r}'\cdot\vec{r} + \vec{r}\cdot\vec{r}'$ by product rule

$= \vec{r}'\cdot\vec{r} + \vec{r}'\cdot\vec{r}$

$= 2\vec{r}'\cdot\vec{r}$

So (1) & (2) are equal, That is,

$0 = 2\vec{r}'\cdot r \Rightarrow \vec{r}'\cdot\vec{r} = 0$

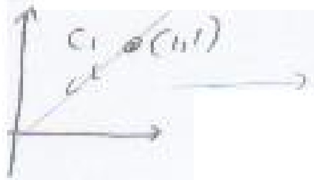
(6) See Last page

thus $\vec{r}'(t)$ is perpendicular to $r(t)$.

(7) COMPUTE LIMITS

(a) $\lim\limits_{(x,y)\to} \frac{x^4+y^4+x^3y}{x^4+y^4}$

C_1]



$C_1 \begin{cases} x = t \\ y = t \end{cases}$

$\lim\limits_{C_1} \frac{x^4+y^4+x^3y}{x^4+y^4} = \lim \frac{t^4+t^4+t^3t}{t^4+t^4} = \frac{3}{2}$

(7.) (CONTINUED)

C_2

$\vec{V} = \langle 1-0, 2-0 \rangle$
$= \langle 1, 2 \rangle$

POINT $= (0,0)$

$x = t$
$y = 2t$

$$\lim_{C_2} \frac{x^4+y^4+x^3y}{x^4+y^4} = \lim_{t \to 0} \frac{t^4+(2t)^4+(2t)^3t}{t^4+(2t)^4}$$

$$= \lim_{t \to 0} \frac{25t^4}{17t^4} = \boxed{25/17}$$

Since $\lim_{C_1} f(x,y) = \frac{3}{2} \neq \frac{25}{17} = \lim_{C_2} f(x,y)$

the $\lim_{(x,y)\to(0,0)} f(x,y)$ D.N.E.

(b) $$\lim \frac{e^{x^2+y^2}-1}{x^2+y^2} = \lim_{r \to 0^+} \frac{e^{r^2}-1}{r^2}$$ POLAR

$$\overset{LH}{=} \lim_{r \to 0^+} \frac{2re^{r^2}}{2r} = e^0 = \boxed{1.}$$

(8.) Let $f(x,y,z) = x\cos(z^2x+y) + x + y^2$

Find TANGENT plane at $P(-1,4,2)$.

$f_x(x,y,z) = 1\cos(z^2x+y) - x\sin(z^2x+y)\cdot z^2 + 1$

$f_y(x,y,z) = -x\sin(z^2x+y)\cdot 1 + 2y$

$f_z(x,y,z) = -x\sin(z^2x+y)\cdot 2zx$

(8) CONTINUED

$f_x(-1,4,2) = \cos(0) - (-1)\sin(0)(2)^2 + 1 = 2$

$f_y(-1,4,2) = +1\cos(0) + 2(4) = 9$

$f_z(-1,4,2) = +1\cos(0) \cdot 2(2)(-1) = -4$

so $\vec{n} = \langle 2, 9, -4, -1 \rangle$

$P = (-1, 4, 2, 14)$

$f(-1,4,2) = -1\cos(0) + -1 + 16$

$= 14$

$$2(x+1) + 9(y-4) - 4(z-2) - (w-14) = 0$$

so

$$w = 2(x+1) + 9(y-4) - 4(z-2) + 14$$

APPROX $f(Q)$ $(Q(-.9, 3.9, 2.1)$ using tangent plane

$$w(Q) = 2(-.9+1) + 9(3.9-4) - 4(2.1-2) + 14$$

$$= +.2 + -.9 - .4 + 14$$

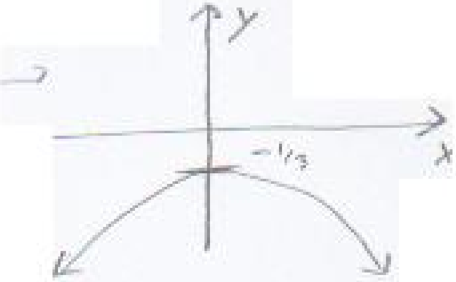
$$= \boxed{12.9}$$

6. GRAPH CONTOUR PLOT for $f(x,y) = x^2 + 3y$.

$z = -1, 0, 1, 2, 3$ ← I shall use these z-values

$\boxed{z=-1}$ $-1 = x^2 + 3y$

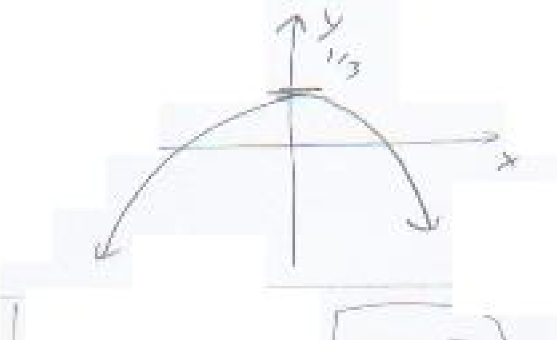
so $y = -\frac{1}{3}x^2 - \frac{1}{3}$



$\boxed{z=0}$ $0 = x^2 + 3y$

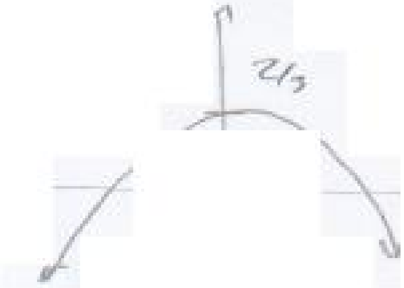
$y = -\frac{1}{3}x^2$

$\boxed{z=+1}$ $y = -\frac{1}{3}x^2 + \frac{1}{3}$



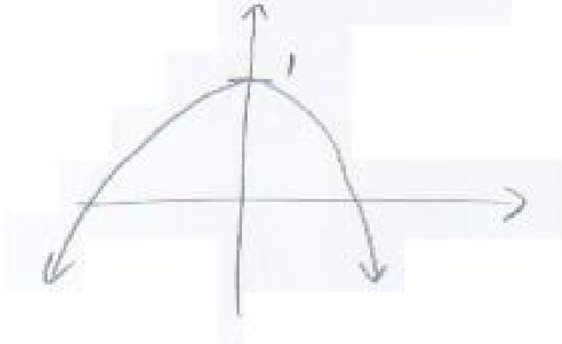
$\boxed{z=2}$

$y = -\frac{1}{3}x^2 + \frac{2}{3}$



$\boxed{z=3}$

$y = -\frac{1}{3}x^2 + 1$



(6) (CONTINUED)