1. (a) $\vec{\nabla} f(x,y,z) = \langle 2xz \; , -2z^2y \; , x^2 - 2zy^2 \rangle$

$\vec{\nabla} f(1,2,-1) = \langle 2(1)(-1) \, , -2(-1)^2(2) \, , (1) - 2(-1)(2)^2 \rangle$

$1 + 8$

$= \langle -2 \; , -4 \; , 9 \rangle$

$D_{\vec{v}} f(1,2,-1) = \vec{\nabla} f(1,2,-1) \cdot \vec{u}$

$= \langle -2, -4, 9 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \rangle$

$= \frac{-6}{\sqrt{2}} = \boxed{-3\sqrt{2}}$

(b) $D_{\vec{v}} f(1,2,-1) = \vec{\nabla} f(1,2,-1) \cdot \frac{\vec{\nabla} f(1,2,-1)}{\|\vec{\nabla} f(1,2,-1)\|} = \sqrt{4 + 16 + 81}$

$= \sqrt{101}$

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2. $f(x,y) = x^3 + 3xy + y^2 - 3$

$f_x = 3x^2 + 3y = 0$ $\quad$ $f_y(x,y) = 3x + 2y = 0$

$y = -x^2$

$3x - 2x^2 = 0$

$x(3-2x) = 0$

$x = 0 \quad 3 - 2x = 0$

$x = \frac{3}{2}$

$P_1 = (0,0) \quad P_2 = (\frac{3}{2}, \frac{-9}{4})$

$D = f_{xx} f_{yy} - (f_{xy})^2 = (6x)(2) - (3)^2 = 12x - 9$

$D(0,0) = -9$	AT $(0,0)$ S.P.
$D(\frac{3}{2}, \frac{-9}{4}) = 12(\frac{3}{2}) - 9 = 9$ $F_{xx}(\frac{3}{2}, \frac{-9}{4}) = \cancel{3(\frac{3}{2})^2 + 3(\frac{-9}{4})} = 6(\frac{3}{2}) = +$	AT $(\frac{3}{2}, \frac{-9}{4})$ MIN

$\nabla f \quad = \lambda \vec{\nabla} g$

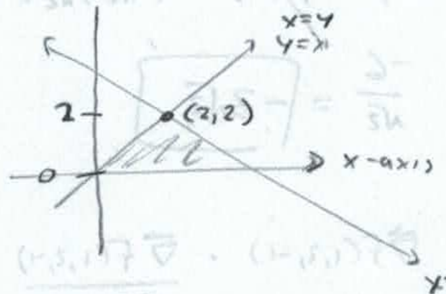
(3) $\langle 2x, 2y, 2z \rangle = \lambda \langle 2, 1, -2 \rangle$

$2x = 2\lambda \quad x = \lambda$
$2y = \lambda \quad y = \frac{1}{2}\lambda$
$2z = -2\lambda \quad z = -\lambda$

$2x + y - 2z = 9$
$2(\lambda) + \frac{1}{2}\lambda - 2(-\lambda) = 9$
$2\lambda + \frac{1}{2}\lambda + 2\lambda = 9$
$\frac{9}{2}\lambda = 9$
$\lambda = 2$

$\boxed{\text{Point } (2, 1, -2)}$

(4.) $\iint_R 3y \, dA$



$y = -3y + 8$
$4y = 8$
$y = 2$

$0 \le y \le 2$

$y \le x \le -3y + 8$

✓

$= \int_0^2 \int_y^{8-3y} 3y \, dx \, dy$

$= \int_0^2 3y \quad x \Big|_y^{8-3y} \, dy$

$= \int_0^2 3y \, [8 - 3y - y] \, dy$

$= \int 3y \, [8 - 4y] \, dy = \int 24y - 12y^2 \, dy = \frac{24y^2}{2} - \frac{12y^3}{3} \Big|_0^2$

$= 12(2)^2 - 4(2)^3 = 48 - 32 = \boxed{16}$

(5.) $\iint 4 [\tan^{-1}(y/x)]^3 \, dA = \int_{\pi}^{3\pi/2} \int_1^2 4 \theta^3 \, r \, dr \, d\theta$

$= \theta^4 \Big|_{\pi}^{3\pi/2} \cdot \frac{r^2}{2} \Big|_1^2 = \left[\left(\frac{3\pi}{2}\right)^4 - \pi^4 \right] \cdot \left[\frac{4}{2} - \frac{1}{2} \right].$

6. $\iint_R \frac{(5x-y)^2}{2x-y}\, dA$

$\begin{cases} u = 5x-y \\ v = 2x-y \end{cases}$

$$\begin{aligned} u &= 5x-y \\ + \; -v &= -2x+y \\ \hline u-v &= 3x \end{aligned}$$

$x = \frac{1}{3}u - \frac{1}{3}v$

$$\begin{aligned} 2u &= 10x-2y \\ -5v &= -10x+5y \\ \hline 2u-5v &= 3y \end{aligned}$$

$y = \frac{2}{3}u - \frac{5}{3}v$

$J(u,v) = \begin{vmatrix} 1/3 & -1/3 \\ 2/3 & -5/3 \end{vmatrix} = -\frac{5}{9} + \frac{2}{9} = -\frac{3}{9} = \underline{\underline{-\frac{1}{3}}}$

L_1: $y = 5x-7$

$y - 5x = -7$

$-u = -7$

$\boxed{u = 7}$

L_2: $y = 5x$

$0 = 5x - y$

$\boxed{0 = u}$

L_3: $y = 2x-2$

$2 = 2x-y$

$\boxed{2 = v}$

L_4: $y = 2x-3$

$3 = 2x-y$

$\boxed{3 = v}$

✓

$= \int_0^7 \int_2^3 \frac{u^2}{v}\left(-\frac{1}{3}\right) dv\, du = -\frac{1}{3} \ln(v)\Big|_2^3 \cdot \frac{u^3}{3}\Big|_0^7$

$= \boxed{-\frac{1}{9}\left[\ln(3) - \ln(2)\right] \cdot \left[7^3 - 0^3\right]}$

$= -\frac{343}{9} \ln\left(\frac{3}{2}\right)$