Math 3330 - Test 1 Review

1 Parametric equations, Polar and Conic Sections

1. Graph the following
   (a) $r = 1 + \cos(\theta)$.
   (b) $r = \cos(\theta)$.
   (c) $r = \cos(2\theta)$.
   (d) $r = \theta$.
   (e) The region inside the circle $r = 2\cos(\theta)$ and out side of the circle $r = 1$.
   (f) The region inside the cardiod $r = 2 + 2\cos(\theta)$ and between $\theta = \pi/2$ and $\theta = 3\pi/4$.
   (g) $x = t + 1$ and $y = t^3 + t$
   (h) $x = 2\cos(t)$ and $y = 2\sin(t)$
   (i) $x = 3\cos(t)$ and $y = 2\sin(t)$
   (j) $x = t\cos(t)$ and $y = t\sin(t)$
2. Compute the tangent lines for the following functions.
   (a) $r = 1 + \cos(\theta)$ at $\theta = \pi/4$.
   (b) $x = t^3 + 3$ and $y = e^{t^3+1}$ at $t = 3$.
3. Compute the areas for the following regions
   (a) $r = 1 + \cos(\theta)$ from $\theta = 0$ to $\theta = \pi$.
   (b) $r = \sqrt{\sin(\theta)}$ from $\theta = 0$ to $\theta = \pi/2$.
   (c) $x = t^3 + 3$ and $y = e^{t^3+1}$ from $t = 0$ to $t = 3$.

2 Vectors and $\mathbb{R}^3$

4. Find the (or a) line that
   (a) contains the two points $A(1, 2, 3, 4)$ and $B(0, 1, 2, 1)$.

(b) contains the point $A(1,2,3)$ and is parallel to $\mathbf{v} = \langle 1, -2, 2\rangle$.

(c) contains the point $A(1,2,3)$ and is perpendicular to $\mathbf{v} = \langle 1, -2, 2\rangle$.

(d) is contained within the plane $x - y + z = 8$

(e) is perpendicular to the plane $x - y + z = 8$

(f) is the intersection of the two planes $P_1 : x + y = 11$ and $P_2 : -1 + 2y - z = 3$.

5. Do the following two lines intersect. If yes where?

$$L_1 : \begin{cases} x &= 1 + 2t \\ y &= 3 - 2t \\ z &= 1 \end{cases} \text{ and } L_2 : \begin{cases} x &= 1 + 5t \\ y &= 3 \\ z &= 1 - 2t \end{cases}$$

6. Find the plane that

(a) contains the three points $A(1,2,3)$, $B(2,3,4)$ and $C(0,-2,1)$

(b) contains the two lines

$$L_1 : \begin{cases} x &= 1 + 2t \\ y &= 3 - 2t \\ z &= 1 \end{cases} \text{ and } L_2 : \begin{cases} x &= 1 + 5t \\ y &= 3 \\ z &= 1 - 2t \end{cases}$$

7. Find the angle

(a) (the acute angle) between the two lines

$$L_1 : \begin{cases} x &= 2t - 5 \\ y &= 3t + 1 \\ z &= 1 \end{cases} \text{ and } L_2 : \begin{cases} x &= 1 + 5t \\ y &= -3 \\ z &= 2t \end{cases}$$

(b) (the acute angle) between the two planes $P_1 : x + y = 11$ and $P_2 : -1 + 2y - z = 3$

(c) (the acute angle) between the line L_1 and plane P_1 from above.

8. Assume we have a block (30 lbs) on an incline of 60° find the two components of the weight vector: perpendicular and parallel to the surface of the incline.

9. Be able to use cross product to compute area and volume.

10. Graph the following (in $\mathbb{R}^3$)

(a) $z = x^2$

(b) $z^2 + x^2 = 4$

(c) the intersection of $z^2 + x^2 = 4$ and $z = 2$

(d) $z = x^2 + y^2$

(e) $z^2 = x^2 + y^2$

(f) $z^3 = x^2 + y^2$

(g) $z^2 = x^2 - y^2$

3 Curves

11. Compute the arc length from $t = 0$ to $t = \pi$ for the function

$$\mathbf{r}(t) = \langle t, \cos(2t), \sin(2t) \rangle.$$

Also compute the displacement from $t = 0$ to $t = \pi$.

12. Graph $\mathbf{r}(t) = \langle t, t^2 \rangle$

13. Graph $\mathbf{r}(t) = \langle 3\cos(t), 2\sin(t) \rangle$

14. For $\mathbf{r}(t) = \langle t^3 + 1, t^2 \rangle$. Compute and then graph the velocity and the acceleration at the points $t = 1$ and at $t = 2$.

15. For $\mathbf{r}(t) = \langle t^3 e^t + 1, t^2 \rangle$. Compute the tangent line at $t = 1$.

16. Prove: $\|\mathbf{v} \times \mathbf{w}\|^2 + (\mathbf{v} \cdot \mathbf{w})^2 = \|\mathbf{v}\|^2 \|\mathbf{w}\|^2$

17. Prove: the product rule for dot product for vector functions

18. Prove: If $\|\mathbf{r}(t)\|$ is constant then $\mathbf{r}(t)$ is perpendicular to $\mathbf{r}'(t)$ at every point t.

4 Functions of one or more variables

19. Graph the following contour plots

(a) $f(x, y) = x^2 + y^2$

(b) $f(x, y) = 5x - 2y$

(c) $f(x, y) = x^2 - y^2$

(d) $f(x, y) = 4 - x^2 - y^2$

(e) $f(x, y, z) = x^2 + y^2 + z^2$

20. Compute limits

(a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^2+2}{x^2+y^2+1}$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^2+xy}{x^2+y^2}$

(c) $\lim_{(x,y)\to(0,0)} \dfrac{e^{x^2+y^2}-1}{x^2+y^2}$

(d) $\lim_{(x,y)\to(0,0)} \dfrac{\cos(x^2+y^2)-1}{(x^2+y^2)^2}$

21. Partial Derivatives

(a) Find the all second order partial derivatives for $f(x,y) = x^2y^3 - e^x$.

(b) Let $f(x,y) = xe^{x^2-y^2} - y + 2$. Find the tangent plane at the point $P(1,-1)$.

(c) Use the tangent plane above to approximate $f(Q)$ for $Q(0.9,-0.9)$.

(d) Let $f(x,y,z) = \sin(zx^2+y) + x + z^2$. Find the tangent plane at the point $P(1,2,-2)$.

(e) Use the tangent plane above to approximate $f(Q)$ for $Q(0.9,1.9,-2.1)$.