**Math 3330 - Test 1**

**Name:**_____

No calculators permitted and show all work. One problem per page.

1. Graph one of the following. Label a few points and the intercepts.
   (a) $r = 2 - \cos(\theta)$. Graph from $\theta = 0$ to $\theta = \pi$.
   (b) $x = 4\cos(t)$ and $y = 2\sin(t)$. Label a few $t$ values and show the directional arrows.
2. Identify the three points $A(0, 2, 1)$, $B(-1, 3, 1)$ and $C(1, -1, 0)$. Find
   (a) Find the plane containing the three points.
   (b) Find the area of the triangle formed by the three points (use the cross product).
   (c) Find one of the angles for the the triangle formed by ABC (tell me which angle you found).
3. Compute the arc length from $t = 0$ to $t = \pi$ for the function
$$\mathbf{r}(t) = \langle t^2, \cos(t^2), \sin(t^2) \rangle.$$
4. Let $\mathbf{r}(t) = \langle 2t^3 + t, t^2 \rangle$.
   (a) Compute and then graph the velocity and the acceleration at the points $t = 1$ and at $t = -1$.
   (b) Compute the tangent line at $t = -11$.
5. Prove: If $\|\mathbf{r}(t)\|$ is constant then $\mathbf{r}(t)$ is perpendicular to $\mathbf{r}'(t)$ at every point $t$.
6. Graph the contour plot for $f(x, y) = x^2 + 3y$.
7. Compute limits
   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 + y^4 + x^3 y}{x^4 + y^4}$
   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{x^2+y^2} - 1}{x^2 + y^2}$
8. Let $f(x, y, z) = x\cos(z^2 x + y) + x + y^2$. Find the tangent plane at the point $P(-1, 4, 2)$. Use the tangent plane above to approximate $f(Q)$ for $Q(-0.9, 3.9, 2.1)$.

9. Assume $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ satisfy the following:

   - $\mathbf{v}_i \cdot \mathbf{v}_j = 0$ whenever $i \neq j$, and
   - $\mathbf{v}_i \cdot \mathbf{v}_i = 1$ for $i = 1, 2, 3$

   **Proposition**: For the vector $\mathbf{w} = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$ we have

   $$\|\mathbf{w}\| = \sqrt{a^2 + b^2 + c^2}.$$

   Prove the proposition. The vectors $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$ are said to be **orthonormal**.