Math 3330 - Test 2 Review

1 Partial Derivatives

1. Be able to graph a gradient on a contour plot.

2. Let $f(x, y) = x^3 - xy\cos(x)$. Compute the gradient of $f(x, y)$.

3. Let $f(x, y) = x^3 - xy$ and let $\mathbf{r}(t) = \langle 3t^2, 2t^3 + 4t\rangle$. Compute the derivative $\frac{df}{dt}$. Use the chain rule.

4. Let $f(x, y) = x^3 - xy$ and let $\mathbf{r}(t, s) = \langle 3s\sqrt{t+1}, t^2 - s^3\rangle$. Compute the derivatives $\frac{\partial f}{\partial t}$ and $\frac{\partial f}{\partial s}$. Use the chain rule.

5. Let $f(x, y) = x^3 - xy$.

   (a) Compute the directional derivative of $f(x, y)$ at $P(1, 2)$ in the direction of $\mathbf{v} = \langle 1, -1\rangle$.

   (b) Compute the directional derivative of $f(x, y)$ at $P(1, 2)$ in the direction of maximum increase.

6. Let $f(x, y, z) = x^2y - z^3$.

   (a) Compute $D_{\mathbf{v}}f(1, 2, 3)$ where $\mathbf{v} = \langle 0, 1, -1\rangle$.

   (b) Compute $D_{\mathbf{v}}f(1, 2, 3)$ in the direction of maximum increase.

7. Compute the extremma of the following:

   (a) $f(x, y) = x^3y^2 - y - x^3$

   (b) $f(x, y) = x^3 - 3x^2 + y^3 - y + 3$

   (c) $f(x, y) = x^3 + x^2y + y^3 - 9y - 3$

   (d) $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x + y - 2z = 1$

   (e) $f(x, y, z) = x + y - 2z$ subject to $x^2 + y^2 + z^2 = 1$

   (f) $f(x, y, z) = x\ln(x) + y\ln(y) + z\ln(z)$ subject to $x + y + z = 1$

2 Double Integrals

8. $\displaystyle\iint_R x + y\,dA$ over the region defined by $x + y = 2$ and the coordinate axes.

9. $\displaystyle\iint_R xy\,dA$ over the region defined by $y = x^2$ and the line $y = x + 1$.

10. $\displaystyle\iint_R e^{x^2}\,dA$ over the region defined by $y = -x$, $y = 2x$ and the vertical line $x = 4$.

11. $\displaystyle\iint_R e^{x^2+y^2}\,dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ in the third quadrant.

12. $\displaystyle\iint_R \sqrt{\frac{\tan^{-1}(y/x)}{x^2+y^2}}\,dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ above the lines $y = -x$ and $y = x$.

13. Find the volume below the paraboloid $z = 12 - x^2 - y^2$ and above the xy-plane.

14. $\displaystyle\iint_R \sin(x-y)\cos(x+y)\,dA$ over the region defined the lines $y = x+2$, $y = x + 4$, $y = -x$ and $y = -x + 3$. Hint the change of variables is $u = x - y$ and $v = x + y$.

15. $\displaystyle\iint_R \frac{x-y}{2x+y}\,dA$ over the region defined the lines $y = x + 2$, $y = x$, $y = -2x + 2$ and $y = -2x + 3$.

16. $\displaystyle\iint_R xy\,dA$ over the region defined the graphs of $xy = 1$, $xy = 3$ and the lines $y = x$ and $y = 3x$. Hint $x = u/v$ and $y = v$.

17. $\displaystyle\iint_R (x-y)e^{x^2-y^2}\,dA$ over the region defined the lines $y = x+2$, $y = x$, $y = -x$ and $y = -x + 3$.

18. $\displaystyle\iint_R e^{x^2+4y^2}\,dA$ over the region defined by the portion of the ellipse $\frac{x^2}{2} + y^2 = 1$ in the third quadrant. Hint use the change of variables $x = 4v\cos(u)$ and $x = v\sin(u)$. And note I had $0 \le u \le 2\pi \ldots$

3 Fields and Line Integrals

19. Compute the following Line Integrals

(a) $\int_C xdy$ where C is the line segment from $(2,1)$ to $(1,2)$.

(b) $\int_C xds$ where C is the line segment from $(2,1)$ to $(1,2)$.

(c) $\int_C \langle x, y\rangle \cdot d\mathbf{r}$ where C is the line segment from $(2,1)$ to $(1,2)$.

(d) $\int_C x - yds$ where C is the upper half of the circle $x^2 + y^2 = 4$ travelling counterclockwise.

(e) $\int_C xydx$ where C is the upper half of the circle $x^2 + y^2 = 4$ travelling counterclockwise.

(f) $\int_C \langle x, 2\rangle \cdot d\mathbf{r}$ where C is the upper half of the circle $x^2 + y^2 = 4$ travelling counterclockwise.

(g) $\int_C xdy$ where C is the portion of the parabola $y = x^2 + 1$ starting at the point $(1,2)$ and ending at $(3,10)$.

(h) $\oint_C \langle x, xy\rangle \cdot d\mathbf{r}$ where C is the entire circle $x^2 + y^2 = 9$ travelling counterclockwise and starting at the point $(3,0)$.

20. Use Green's Theorem if possible to solve the following. If not possible to use Green's theorem explain why.

(a) $\oint_C \langle x, -y\rangle \cdot d\mathbf{r}$. Let C be outside of the square traced from $(0,0)$ to $(0,2)$ to $(1,2)$ to $(1,0)$ and then back to $(0,0)$.

(b) $\oint_C \langle e^{x^3} - xy, e^{y^3} - y\rangle \cdot d\mathbf{r}$. Let C be outside of the triangle traced from $(0,0)$ to $(0,2)$ to $(1,2)$ and then back to $(0,0)$.

(c) $\oint_C \langle \cos(x^2) + y, \cos(y^2) + xy\rangle \cdot d\mathbf{r}$. Let C be the circle $x^2 + y^2 = 4$ traced counter-clockwise.

(d) $\int_C x^3dx$ where C is the upper half of the circle $x^2 + y^2 = 4$ travelling counterclockwise.