**MA 2310: Practice Test 2**

# 1 Related Rates

1. Assume a spherical balloon is being blown up with a rate of $300in^3/s$. When the radius is 4 inches how fast is the radius increasing?

2. Assume a spherical balloon is being blown up with a rate of $300in^3/s$. When the radius is 8 inches how fast is the radius increasing?

3. Assume a spherical balloon is being blown up with a rate of $300in^3/s$. When the radius is 8 inches how fast is the surface area increasing?

4. Assume a spherical balloon is being blown up and the radius is increasing at a rate of $0.5in/sec$. How fast is the volume increasing when the radius is 8?

# 2 Graphing 4.1-4.3

5. Find and classify the extrema for

   (a) $f(x) = x^3 - 12x + 1$

   (b) $f(x) = x^{5/3} - x^{2/3}$

   (c) $f(x) = x^4 + 8x^3 - 270x^2 + 1$

   (d) $f(x) = (x+1)^2(2x-5)^3$

   (e) $f(x) = e^{x^3-3}$

   (f) $f(x) = \frac{x^2}{x-2}$

   (g) $f(x) = x^3 - 12x + 1$

   (h) $f(x) = \sqrt{x}\ln(x)$ where $x > 0$

   (i) $f(x) = 2\tan^{-1}(x) - x$

6. Find the 1st derivative and 2nd derivative number lines. List the intervals where $f(x)$ is increasing, decreasing, concave up and concave down.

   (a) $f(x) = x^3 - 12x + 1$

   (b) $f(x) = x^{5/3} - x^{2/3}$

   (c) $f(x) = x^4 + 8x^3 - 270x^2 + 1$

   (d) $f(x) = (x+1)^2(2x-5)^3$

   (e) $f(x) = e^{x^3-3}$

   (f) $f(x) = \frac{x^2}{x-2}$

   (g) $f(x) = x^3 - 12x + 1$

   (h) $f(x) = \sqrt{x}\ln(x)$ where $x > 0$

   (i) $f(x) = 2\tan^{-1}(x) - x$

7. Graph the following. Find and classify the extrema; label on graph. Find the 1st derivative and 2nd derivative number lines. Identify each critical point on the graph and the points of inflection.
   (a) $f(x) = x^3 - 12x + 1$
   (b) $f(x) = x^{5/3} - x^{2/3}$
   (c) $f(x) = x^4 + 8x^3 - 270x^2 + 1$
   (d) $f(x) = (x+1)^2(2x-5)^3$
   (e) $f(x) = e^{x^3-3}$
   (f) $f(x) = \frac{x^2}{x-2}$
   (g) $f(x) = x^3 - 12x + 1$
   (h) $f(x) = 2\tan^{-1}(x) - x$

## 3 Applications: 4.4 - 4.8

8. A farmer wishes to build a rectangular fenced in pen with 100 linear feet of fencing. She wishes to maximize the area. What are the dimensions of maximum area?

9. A farmer wishes to build a rectangular fenced in pen with 2500 square feet of area. She wishes to minimize the amount of fencing used to build the pen. What are the dimensions to minimize the amount of fencing used?

10. The farmer now is going to build a fenced in pen along the the side of the barn. So the farmer only needs to build three sides of the pen (the barn will serve as the fourth side of the pen). Again she has 300 linear feet of fencing and she wishes to maximize the area. What are the dimensions of maximum area?

11. Our busy farmer is going to build one more pen for her horses. One side of the pen is alongside a roadway and fencing along roadways costs more then the fencing for the other three sides will cost. Along the road the fencing will cost \$3.00 per linear foot; while the fencing on the other three sides will only cost \$2.00 per linear foot. She knows that her horses will be only happy with an area of at least 3000 square feet of fencing. She wishes to build the fence for minimum cost. What are the dimensions of minimum cost?

12. A cracker company wants to build a saltine cracker box with minimum surface area for their 128 fl ounce box of crackers (note 128 ounces = 231 cubic inches). The box is to be a rectangular prism and to be so that the height is twice length of the base. Find the dimensions with minimum surface area.

13. Now my oatmeal maker wants to hire us to design a new oatmeal container. The oatmeal containers are cylinders with a circular base. Assume we have 100 square inches of cardboard to build each oatmeal container. What are the dimensions of the container with maximize volume?

## 4 Linear approximation and differentials

14. Find the linear approximation of $f(x) = x/(x+3)$ at $a = -2$. That is find $L(x)$.

15. Find the linear approximation of $f(x) = 3x^2 \ln(x) - x$ at $a = 1$. That is find $L(x)$. Now use to compute $L(0.97)$.

16. Find the linear approximation of $f(x) = \sin(x)$ at $a = 0$. Now use to compute $L(0.02)$.

## 5 Mean Value Theorem

17. apply MVT for $f(x) = x^3$ on interval for $[-3, 3]$, that is find c

18. apply MVT for $f(x) = |x|$ on interval for $[-3, 3]$, that is find c

19. apply MVT for $f(x) = x^2$ on interval for $[-3, 3]$, that is find c

20. Find the following derivatives

    (a) $y = x^x$

    (b) $y = [\sin(x)]^x$

    (c) $y = x^{\sin x}$

    (d) Find the line tangent to $y = x^{x^2}$ at $x = 1$.

21. Let $s(t) = 3t^2 - 2t + 1$ represent the position of a particle.

    (a) Find the position, velocity and accelerration of the particle at time $t = 0$.

    (b) At what time does the particle stop moving?

22. Let $s(t) = -16t^2 + 20t + 10$ represent the height of a ball thrown into the air. the ball is thrown at time $t = 0$.

    (a) What is the height of the ball at time $t = 0$?

    (b) At what time does the ball stop moving? What is the height of the ball at this instant?

    (c) When does the ball hit the ground?

## 6 L'Hôpital's Rule

23. $\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$

24. $\lim_{x\to 0} \frac{sin(3x)}{7x}$

25. $\lim_{x\to 0} \frac{\sin(\sin(x))}{x}$

26. $\lim_{x\to 4} \frac{\frac{1}{2} - \frac{1}{\sqrt{x}}}{x-4}$

27. $\lim_{x\to 0^+} \frac{1}{x} - \frac{1}{\sqrt{x}}$

28. $\lim_{x\to 0^+}$

29. $\lim_{x\to\infty} \frac{x^2+1)}{x}$

30.

31. $\lim_{x\to 0} \sin(x)x^{-1/2}$

32. $\lim_{x\to 0} x\ln(x)$

33. $\lim_{x\to\infty} x\sin(\frac{1}{x})$

34. $\lim_{x\to 0}(1-3x)^{1/x}$

35. $\lim_{x\to 0} x^x$

36. $\lim_{x\to\infty}(1+\frac{1}{x})^x$

37. Find the Antiderivatives for the following:

    (a) $\int x^2 + 1\,dx$
    (b) $\int x(x^2+1)\,dx$
    (c) $\int \frac{x^2+1}{x}\,dx$
    (d) $\int \frac{1}{1+x^2}\,dx$
    (e) $\int 3\cos(x) - \sec^2(x)\,dx$
    (f) $\int e^x - \frac{2}{x} + \frac{7}{2x^2}\,dx$
    (g) $\int \frac{\sin(x)}{\cos(x)}\,dx$

38. Find the Antiderivatives for the following, using substitution.

    (a) $\int 2xe^{x^2+1}\,dx$
    (b) $\int 2x\sin(x^2+1)\,dx$
    (c) $\int 2x\sec^2(x^2+1)\,dx$
    (d) $\int x^2\sin(x^3+1)\,dx$
    (e) $\int x^2\sec(x^3+1)\tan(x^3+1)\,dx$
    (f) $\int \frac{x}{x^2+1}\,dx$
    (g) $\int \frac{\sqrt{\ln(x)+11}}{x}\,dx$