MATH 5320 Test 2

Name:_____

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 5$. Prove f is continuous (using the $\varepsilon - \delta$ definition).

2. Define $f(x) = \begin{cases} \frac{x^4+3x^3}{|x|} - 2x^3 + 1 & : x \neq 0 \\ 1 & : x = 0 \end{cases}$.

   (a) Show f is continuous at $x = 0$.

   (b) Compute $f'(0)$ if it exists.

3. State and prove only **one** of these theorems: the product rule for differentiation or the FTC, v1.

4. We say a function is Lipshitz k if there is some $k > 0$ so that

$$|f(x) - f(y)| \leq k|x - y| \text{ for all } x, y \in \mathbb{R}.$$

   (a) Prove if f is Lipshitz k then f is uniformly continuous.

   (b) Prove if $|f'(x)| \leq 1$ for all $x \in \mathbb{R}$ then f is Lipshitz k. (Hint use the MVT).

5. For Taylor's theorem with $n = 3$, find a polynomial to approximate $f(x) = \cos(2x)$ at $a = 0$ over the range $[a, b] = [0, 0.1]$. Bound the remainder for the Taylor series.

6. Mean Value Theorems

   (a) State the MVT. And what does the MVT say for the function $f(x) = |x| + 1$ over the interval $[-2, 4]$? Find any c satisfying the MVT.

   (b) State the MVTI. And what does the MVTI say for the function $f(x) = |x| + 1$ over the interval $[-2, 4]$? Find any c satisfying the MVTI.

7. Let $f : [0, 5] \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 2 & : 0 < x < 5 \\ -1 & : x = 0 \text{ or } x = 5 \end{cases}$.

   (a) Graph $f(x)$.

   (b) Prove using Riemann Sums that $f(x)$ is integrable. What is the integral, I?