MATH 5320 Test 1

Name:_____

1. Define the following:

   (a) $f : A \to B$ is injective $\iff$

   (b) $\alpha = \sup A \iff$

   (c) $A \sim B \iff$

   (d) $\lim_{n\to\infty} a_n = a \iff$

2. Let the set A be nonempty and bounded above and let $\alpha = \sup(A)$. Show for all $\varepsilon > 0$ there is some $x \in A$ so that $\alpha - \varepsilon < x \leq \alpha$.

3. Show $\mathbb{Q}$ is countable.

4. Do **one** of the following:

   - Prove using the $\varepsilon - N$ definition that

   $$\lim_{n\to\infty} \frac{2n+1}{3n+1} = \frac{2}{3}$$

   - Prove using the $\varepsilon - N$ definition that

   $$\lim_{n\to\infty} \frac{n^2+1}{3n+1} = \infty$$

5. Do **one** of the following:

   - If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then show $\lim_{n\to\infty} a_n b_n = ab$.
   - If $\lim_{x\to c} f(x) = F$ and let $k \in \mathbb{R}$ then show $\lim_{x\to c} kf(x) = kF$.

6. Prove using the $\varepsilon - \delta$ definition that

$$\lim_{x \to -2} x^2 + x - 1 = 1.$$